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Portfolio Allocation and Optimization with Carbon Offsets: Is it Worth the While?

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Abstract

We explore whether the integration of carbon offsets into investment portfolios improves performance. Our results show that investment strategies that include such offsets achieve higher Sharpe Ratios than the diversified benchmark portfolios. The efficient frontier of optimal portfolio choices is shifted upwards as a result of including compliance and voluntary carbon offsets in the portfolio. Our results also show that while diversified portfolios may benefit from carbon offsets integration, voluntary carbon offsets are significantly more sensitive to exogenous shocks than compliance carbon allowances. All these results are novel and may encourage investors to invest in such sustainable asset classes.

JEL Classification: G11, G12, Q21, Q23, Q54
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1. Introduction

The global volume of voluntary carbon offsets resulting from avoided deforestation, afforestation or renewable energies, among others, is expected to grow from approximately USD 1 bn. in 2021 to USD 50 bn. in 2030, according to most recent estimates (Blaufelder et al., 2021). Such an exponential growth potential will likely attract new investors to a market which has traditionally been dominated by international companies with voluntary net-zero goals, foundations, philanthropists, and governments, mostly from developed countries. In comparison, the compliance carbon markets around the world are huge, the largest being the European Emissions Trading Scheme (EU ETS).[1] The goal of this study is to analyze whether it is worth the while investing in compliance allowances and voluntary carbon offsets[2] from a risk-return perspective, making use of the publicly available EU ETS pricing data and the scarce publicly available pricing and return data for voluntary carbon offsets.

We document that when compliance allowances and voluntary carbon offsets are included in asset portfolios and in portfolio optimization strategies, the risk-return profile of the analyzed investment strategies improves. Significantly higher Sharpe Ratios are obtained when compliance allowances and voluntary carbon offsets are part of the asset allocation mix that includes all relevant traditional asset classes. These results, which are documented for the first time in the literature, are likely to attract more and new investors such as purely financially motivated or retail investors especially to voluntary carbon markets. This, in turn, may result in more fund flows into these markets and, hence, ultimately lead to more environmental protection thereby making a significant contribution to the fight against climate change.

Nordhaus (1977; 1991; 1992), among others pioneered the idea of incorporating climate risks into macro analyses. Traditionally, asset and investment managers focused on determining intrinsic

[1] Global compliance carbon markets reached a size of USD 851 bn. in 2021, with the EU ETS accounting for the lion's share of USD 769 bn. (Refinitiv, 2022).
[2] We use the term 'carbon offsets' as to classify voluntary carbon offsets as carbon credits for the purpose for offsetting emissions while compliance allowances are permits or certificates backed by regulators for the permission to pollute or the purpose of 'offsetting' regulated emissions.

and extrinsic values of financial and physical assets without a major focus on internalizing, for instance, human-driven carbon emissions. Nowadays, there is wide agreement about the reality of climate change and its impact on people's way of life. Human-driven activities account for most of the ongoing climate change and pose large aggregate risks to the economy and the global financial system (Behnam et al., 2020). To maintain a smooth consumption stream, however, investors aim to determine the sensitivity of each component of the overall portfolio to changes in asset returns in what can be called *the effective asset mix.* Asset class factor models provide intuitive ways of observing the sensitivities of portfolio returns to each risk factor; modeled for (1) mutual exclusivity, (2) exhaustivity, and (3) 'differing' returns (Sharpe, 1992). However, most crucial to asset allocation is the construction of portfolios that offer attractive risk-reward characteristics.

Very little is documented in the literature about the contribution of compliance allowances and especially voluntary carbon offsets to portfolio performance and optimal asset allocation in general. This study provides first empirical evidence addressing this open research question. The central objective of this paper is to understand and compare the performance of portfolios that integrate compliance allowances and voluntary carbon offsets versus those that do not. We construct several combinations of optimal portfolios under the mean-variance framework and measure portfolio performance using Sharpe Ratios by integrating compliance allowances and voluntary carbon offsets into a benchmark asset portfolio of bonds, commodities, currencies, equities, lumber and REITS. To gauge portfolio outperformance, we conduct in-sample-out-of-time tests on the basis of optimal portfolio weights constructed from a rolling sample window of the last 90 days. We construct an inverted studentized bootstrap of the difference in Sharpe Ratios from the different combinations of portfolios to test for the significance of Sharpe Ratio differences and reject the null of no statistical difference in Sharpe Ratios by applying inference methods outlined in Ledoit and Wolf (2008).

Our results mostly show that portfolios that include the compliance allowances and voluntary carbon offsets outperform those that do not both in-sample and in-sample-out-of-time. In particular, the inclusion of compliance allowances and voluntary carbon offsets proxied by the European Emissions Allowances (EUA), Global Emission Offset Futures (GEO) and Nature-based GEO (NGEO) significantly increases portfolio Sharpe Ratios. In fact, we show that including EUA, GEO and/or NGEO in the benchmark asset portfolio causes the efficient frontier of optimal portfolio choices to be shifted upwards for the in-sample analyses.

Additionally, GEO and NGEO exhibit properties unique to independent asset classes: i) low external correlations, ii) significant internal correlations, and iii) differing returns. Conte and Kotchen (2010) show that price discovery in voluntary carbon markets is uniquely driven by project characteristics which may be economically reinforced by buyers. We argue that unstandardized price discovery and non-mandatory participation are fundamental to voluntary carbon markets´ design (versus, e.g., compliance carbon markets or other asset classes) and remains a key reason for the low external correlations.

To gain further insight into the risk-return properties of the emissions products we study, we explore their volatility and return spillover behaviors in a network system. We observe that for most of the portfolios we construct, carbon products are volatility and return recipients. This empirical result seems consistent from a macro perspective as net carbon emissions are major by-products of human economic activity. In particular, we identify volatility spillover patterns from some asset classes (bonds in particular) which feature prominently with GEO and NGEO but not with EUA. As bond markets are broadly accepted to be a key indicator for the health of the global economy, the spillover results suggest GEO and NGEO may be sensitive to exogenous shocks tied to the economy.

On the contrary, we find EUA to be more resilient to volatility spillovers from bonds. In fact, EUA tends to be a marginal transmitter of volatility in the networks we study. While this result does not exclusively absolve EUAs from exogenous shocks tied to the economy, it may be explained by the fact that more and more companies are beginning to consider the cost of emissions as a balance sheet item, thereby making EUA a potential volatility transmitter in a network. We find this conjecture to be confirmed in in-sample out-of-time analyses where optimal rolling GEO and NGEO weights drop-off significantly at the onset of the war between Russia and Ukraine. These results, which, to the best of our knowledge, are documented for the first time in the literature, provide empirical evidence that in the fight against climate change, voluntary abatements may become a secondary priority in the presence of large negative and asymmetric shocks to the global economy.

While the literature on portfolio optimization with compliance allowances and voluntary carbon offsets is scarce, the literature on the broader field of climate finance is growing. Jagannathan and Ma (2019) argue that investors can reduce overall portfolio risk by integrating sustainability criteria into the investment process. Rameli et al. (2021) show that investors react to political events related to firms' climate response strategies. Heinkel et al. (2001) show how divestment from high carbon emission companies may result in higher stock returns. Hong et al. (2019) explore how climate risk is priced into equity markets. Hsu et al. (2022) provide a framework that shows how high polluting firms are more exposed to environmental regulation risk and command higher risk premiums. Engle et al. (2020) argue that dynamic portfolio management strategies of climate risk may be pursued by extracting actionable investment intelligence from a Wall Street Journal-based climate news index. Choi et al. (2020) show that carbon-intensive firms´ equity prices underperform during times with abnormally warm weather. The results of our study add to this literature by documenting for the first time the benefits of including compliance allowances and voluntary carbon offsets into asset allocation and portfolio optimization.

The rest of this paper is organized as follows. Section 2 presents an overview of carbon markets. Section 3 presents all data used in this paper and the empirical approach used in the empirical analyses. All results are presented in Section 4 and conclusions are outlined in section 5.

2. Carbon markets

Carbon markets may be broadly classified into compliance and voluntary carbon markets. Compliance carbon markets have historically garnered lots of attention, providing agents with real options to pollute backed by regulators. The largest compliance carbon market, the EU ETS accounts for about 90% of the global compliance carbon market. In terms of market design, the EU ETS is made up of primary and secondary markets. The primary market organises auctions where agents directly purchase certificates at a minimum price set by the regulator. The secondary market allows demand and supply interactions between agents at organised marketplaces and provides support for prices. Surplus positions provide a market for speculators who pursue shorter-term small-scale investments to arbitrage price changes, thus providing liquidity and reducing short-term fluctuations in the market (Schopp et al., 2015). Arbitrage is mostly pursued by banks. Since EUA certificates are bankable at zero cost, banks procure significant amounts and sell them forward. When agent interactions do not yield sufficient demand-side price support, the regulatory toolbox includes supply-side instruments for achieving price support. In 2014, the European Commission designed a Market Stability Reserve (MSR) in a bid to reduce the surplus of emissions certificates. These unilateral market interventions are triggered by the breach of an upper tolerance limit of emissions surpluses. If surpluses fall below a predefined lower trigger, some allowances are released from the reserve.

The major difference between compliance and voluntary carbon markets is in their market design, operation and price discovery. Voluntary carbon markets offer participants the promise of a direct reduction, removal or avoidance of carbon emissions underlined by a carbon project. The value of voluntary carbon offsets may account for a variety of factors such as project type, location and buyer preferences, amongst others.[3] This unique market design has direct implications for price discovery which is murkier than in standard traded commodities, but also ensures that voluntary offsets are different from equities, cash or bonds. Hence and crucially, price discovery is intrinsically determined by project characteristics (Conte and Kotchen, 2009).

GEO offers physical settlement of carbon offsets from three different registries as underlying.[4] GEO-based certificates meet the eligibility criteria for the Carbon Offsetting and Reduction Scheme

[3] Project prices are a function of budgetary allocations. Economically, intrinsic carbon value would be at least equivalent to drivers of GHG emissions.
[4] These are Verified Carbon Standard, American Carbon Registry and Climate Action Reserve.

for the International Aviation (CORSIA). Similar to GEO, N-GEO offers the possibility for corporations to offset net carbon emissions using nature-based agriculture, forestry and other land use projects (AFOLU). N-GEO projects are registered under the Verified Carbon Standard (VCS) and Climate Community and Biodiversity Standards (CCB) labels. GEO and N-GEO bring more transparency to the price discovery process in the voluntary market and offer risk and portfolio managers new tools for managing net emission exposures and risks.

3. Data and investment strategies

This sutdy assesses the performance of portfolios that integrate compliance allowances and voluntary carbon offsets versus those that do not. We construct three categories of portfolios from a set of established asset classes and emissions-based products under the mean-variance paradigm, subject to different optimization constraints. Table 1 presents the assets considered in this paper and their respective proxies. Category 1 portfolios are constructed using data from 7th October, 2017 – 13th October, 2022. Category 2 portfolios comprise the time period 1st March, 2021 – 13th October, 2022. For category 3, we include prices from 4th August, 2021 – 13th October, 2022 for the respective assets. We obtain daily close of business information for all assets from BarChart, DataStream and NASDAQ for the period October, 2017, to October, 2022.

The selection of traditional asset classes was guided by considerations such as being able to gain exposure to asset classes generally available to reasonably sophisticated investors. While ignoring single assets within asset classes, whenever available, we opted for investable assets, mostly Exchange Traded Funds (ETFs). The ETFs selected track indices that broadly represent the asset classes. Hence their performances are intrinsically linked to their underlying construction and are replicable with assets within each asset class. Additionally, the ETFs function as plausible retail proxies for asset classes providing wide exposure within each traditional asset class and the chance to explore performance benefits of integrating emissions-based products with more established asset classes. Considering that a major by-product of all human activity is carbon-related emissions, we include commodities (S&P GSCI Commodities) and lumber. Foresters and landowners are constantly faced with the choice of either felling trees or preserving forests. The inclusion of lumber thus represents value gained from alternative land usage other than preservation and captures purely financial incentives and preferences.

Figure 1 visualises the universe of assets considered in this paper. All prices are indexed to the initial prices in order to visualise their respective price evolution over the period. Compliance carbon, proxied by the EUA, is most dominant in Figure 1 as it rises by over 2.5 times above its initial level during the period. Similarly, GEO, NGEO and lumber experience boom periods, albeit to lesser extents in that order. We observe the unanimous drop in all asset prices at the onset of the COVID-19 pandemic in 2020 and the general downward trend in price growth at the beginning of 2022, for instance, because of the Russia-Ukraine war and inflation. In fact, EUA, GEO and NGEO prices experience sharp declines at the beginning of the war on 24th February, 2022.

Figures 2-4 present significance tests for the correlation relationships between the assets within the different time periods. Figure 2 presents results of correlation significance tests of returns between asset pairs masking out correlation pairs that miss conventional significance. Portfolios formed from the dataset in Figure 2 fall under category 1 portfolios. We observe statistically significant correlation between EUA returns and all assets in the dataset over this 5-year period. Considering that emissions are a by-product of most economic activities, the value of EUA may have both short and long run relationships[5] with the assets in our universe. Figures 3-4 integrate GEO and NGEO returns into the correlation matrix. For category 2 data, GEO is statistically uncorrelated with any of the other asset classes. In Figure 4, we observe that NGEO, which offers a means to achieve emissions-reduction targets using high-quality, nature-based offsets sourced exclusively from agriculture,

[5] Gronwald et al. (2011) document a positive dependence structure between EUA and coal, gas and power futures and EUA versus equity spot returns. Chevallier (2009) finds weak forecasting evidence of EUA futures using equity dividend yields and the junk bond premium. Chevallier (2011) finds time-varying interactions between EUA, macroeconomic variables and commodities while Chen et al. (2019) show a stable yet time-varying correlation between a variety of energy commodities and the EUA.

forestry, and other land use (AFOLU) projects, has a statistically weak negative correlation with commodities and, more importantly, a moderately positive correlation with GEO. GEO and NGEO exhibit moderate internal but low external correlations in a result that is underlined by the murky price discovery process in voluntary carbon markets.

We do not observe any significant correlation between GEO/NGEO and EUA which is initially surprising. This may be due to the structural differences in their market design. Behr et al. (2022) show empirically that compliance markets respond more strongly than voluntary carbon markets to climate policy events, partly by design. Appendix Figures 1-3 provide further evidence of the time-varying correlations between the assets and EUA, GEO and NGEO and the rather low external correlations between GEO and NGEO and other asset classes across the three data categories based on 250- and 60-day rolling windows respectively.

To estimate returns, we refrain from using historical means as these have been shown to be rather uninformative in the literature. They tend to reinforce estimation error-maximization and spread these errors in the expected returns vector (Lee, 2000; Michaud, 1989). Several studies establish the theoretical and empirical inadmissibility of historical returns for portfolio optimization[6] while others[7] develop and test various Bayesian and non-Bayesian solutions for extracting risk-adjusted expected returns using various extensions of the capital asset pricing model (CAPM).

In this paper, we back out risk-adjusted expected returns using the well-known capital asset-pricing model (Sharpe, 1964). In the absence of an appropriate market reference, we construct an equally-weighted market portfolio from the asset universe for each category from which expected risk-adjusted returns are backed out. In the absence of a sufficiently long time series history that bakes in several business and economic cycles, backing out risk-adjusted returns for each asset captures the expected returns for which investors are willing to hold the assets in our universe and provides a reasonable equilibrium reference.

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[6] See Jorion (1991), Berger and Bock (1976), Efron and Morris (1973; 1975; 1976), Michaud (1989; 2019).
[7] See Mynbayeva et al. (2021) Jorion (1991); Black and Litterman (1992); Esmaeili et al. (2021).

For the variance-covariance matrix, we apply the Ledoit and Wolf (2004) shrinkage methodology to reduce the kind of estimation error likely to perturb a mean-variance optimizer[8]. The operational shrinkage estimator of the covariance matrix Σ is given by:

$$\hat{\Sigma}_{shrink} = \widehat{\delta^*}F + \left(1 - \widehat{\delta^*}\right)S$$

where $0 \leq \delta^* < 1$ is a shrinkage constant that minimizes the expected distance between the shrinkage estimator and the true covariance matrix.[9]

With $\hat{\mu}$ and $\hat{\Sigma}_{shrink}$ known, we are able to estimate portfolio weights W on the basis of the investors' utility function in equation (2). Finally, to quantify portfolio performance, we compute Sharpe Ratios for each constructed portfolio strategy as:

$$SR = \frac{\hat{\mu} - r_f}{\hat{\delta}} \qquad (4)$$

where $\hat{\mu}$ is the calculated portfolio return, r_f the risk-free rate of return[10], and $\hat{\delta}$ the portfolio standard deviation.

Table 2 presents the respective number of observations (N), the CAPM-based expected returns E[μ], and the Ledoit-Wolf standard deviations LW[σ] backed out of the shrunk covariance matrix for the three categories of data used in this paper. Panel A comprises EUA and the six assets – bonds, commodities, currencies, equities, lumber, and REITS – which we refer to as the benchmark portfolio. EUA has the highest expected risk-adjusted return (0.267) and volatility (0.454), just ahead of lumber. Panel B comprises all assets listed in Panel A but includes GEO. Here, GEO has the second highest mean (0.245) and volatility (0.561) after lumber, while the EUA profile of high expected returns (0.224) and volatility (0.495) is also validated in this period. Panel C includes all assets listed in Panel B and adds NGEO. Compared to other assets in our asset universe, investors require higher returns

[8] There is an extensive literature on the Bayesian approach to estimation error some relying on diffusion priors. These include Barry (1974) and Bawa et al. (1979). As regards shrinkage estimators, confer to Jobson and Korkie (1981) or Memmel (2003). In terms of asset pricing priors cf. Pástor, (2000) or Pástor and Stambaugh, (2000), among others.

[9] We apply a constant variance shrinkage, i.e., target $\delta^* = \mu(\sigma_{11}^2, \ldots, \sigma_{jj}^2)$ is the mean of asset variances on the diagonal and zero elsewhere $\begin{bmatrix} {\sigma^2}_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & {\sigma^2}_{jj} \end{bmatrix}$.

[10] Set at 2%.

to hold EUA, GEO, NGEO and lumber, compensating for the higher volatility associated with these assets. On the contrary, we observe that bonds report the lowest risk-adjusted expected returns and volatility, accounting for investors' perception of a low default probability of U.S. Government bonds. All assets have positive risk-adjusted expected returns across all data categories.

Table 3 provides insights into the covariance matrix across the three categories. The parameter estimates provided represent the shrunk covariance matrix from Ledoit and Wolf (2004) covariance. An important characteristic of diversified portfolios is their low covariances with other assets in the portfolio. We observe that across all categories, the established benchmark assets exhibit low covariances with each other.[11] EUA, GEO and NGEO also report low covariances with the benchmark assets.[12] Assets with high expected returns and low or negative covariances with other assets are usually good components of diversified portfolios. EUA, GEO and NGEO seem to fit these characteristics well in the sample period.

3.1 Asset volatility and return spillovers

Volatility and return spillovers provide information about the connectedness of markets. Their importance for portfolio optimization and applications in finance have been increasingly discussed in the literature (see Diebold and Yilmaz, 2009, 2012; Dahl and Jonsson, 2018; Kang et al., 2017; Lucey et al., 2014; Katsiampa et al., 2019; Corbet et al., 2020). Crucially, spillovers estimate the share of new information not fully priced into an asset. Hoang and Baur (2021) show that explicitly considering volatility and return spillovers is not necessary for asset allocation as spillovers are already incorporated in contemporaneous correlations of returns and volatility in the determination of optimal portfolio weights. In practice, however, spillovers are intuitive and important for portfolio managers to understand interdependencies and to identify the origin and impact of spillovers. We apply the

[11] Off-diagonals: Panel A, min/max: -0.003/0.034, Panel B, min/max: -0.012/0.028, Panel C, min/max: -0.019/0.032.
[12] Off-diagonals: EUA min/max: -0.005/0.027; GEO min/max: -0.012/0.111; NGEO min/max: -0.019/0.111. COV(GEO, NGEO) = 0.111 is the highest covariance result and synonymous with high internal correlation.

Diebold and Yilmaz (2012) framework based on the general forecast variance decomposition introduced by Pesaran and Shin (1998) to construct volatility and return spillovers for the respective time periods in our sample. Specifically, for a covariance stationary N-variable VAR(p), $x_t = \sum_{i=1}^{p} \phi_i x_{t-i} + \varepsilon_t,\ \ where\ \varepsilon \sim (0, \Sigma)$ is a vector of i.i.d. disturbances. The moving average representation is given by $x_t = \sum_{i=1}^{\infty} A_i \varepsilon_{t-i}$, where for the NxN coefficient matrices $A_i = 0\ for\ i < 0$. The variance decompositions allow us to assess the fraction of the H-step-ahead error variance in x_i forecasts due to shocks to $x_j, \forall\ j \neq i$ for each i. We can write the directional volatility (return) spillover received by asset i *from* all other assets j as:

$$S_{i.}^{g}(H) = \frac{\sum_{\substack{1 \le j \le N \\ j \neq i}} \widetilde{\theta_{ij}}(H)}{N} * 100$$

Similarly, the directional volatility (return) spill-overs from asset j *to* all other assets i may be written as

$$S_{.i}^{g}(H) = \frac{\sum_{\substack{1 \le j \le N \\ j \neq i}} \widetilde{\theta_{ji}}(H)}{N} * 100$$

Accordingly, net spillover from asset i to all other assets may be written as

$$S_{i}^{g}(H) = S_{.i}^{g}(H) - S_{i.}^{g}(H)$$

To facilitate x portfolio construction tests including EUA, GEO and N-GEO, we stratify the dataset into three data categories as previously explained, the latter two on the basis of when data are available for GEO and NGEO.

3.2 Asset allocation portfolios

We aim to build broad, diversified and stable portfolios that provide the best risk-return performance. Generally, we consider our objective and constraints to be convex as is standard in the finance literature. What follows is a description of the framework within which we construct all portfolios. The list of strategies includes Naïve (1/N) diversification, a CAPM strategy which we split into Long Only

Strategy and Long-Short Strategy for in-sample out-of-time tests and the Minimum Volatility strategy.

We denote with R_t the N-vector of risk-adjusted returns on the N risky assets; μ_t is an Nx1 vector of risk-adjusted expected returns from a CAPM model with an equally-weighted market benchmark constructed from all assets in the portfolio, and Σ_t, the Ledoit-Wolf shrunk variance covariance matrix of dimension NxN. Let their estimated versions be called $\hat{\mu}_t$ and $\hat{\Sigma}_t$ respectively. Also, let T be the length of the data series and $\mathbf{1}_N$ an N-dimensional unit vector. Finally, W is an Nx1 vector of invested portfolio weights in the N assets. For the avoidance of doubt, the normalized portfolio weights with directional orthogonality preserved is represented as:

$$W_t = \frac{w_t}{|\mathbf{1}_N^T w_t|} \quad (1)$$

To enable within-strategy portfolio result comparisons, we consider a standard mean-variance investor who has access to the asset universe discussed above. Typically, the portfolio manager chooses W to maximize expected utility[13]:

$$\max_W w_t^T \mu_t - \frac{\gamma}{2} w_t^T \Sigma_t w_t \quad (2)$$

where γ may be considered as investor risk aversion and the solution to the optimization problem

$$w_t = \frac{1}{\gamma} \Sigma_t^{-1} \mu$$

Hence, the vector of weights invested within the portfolio of N assets at time t may be characterized as

$$W_t = \frac{\Sigma_t^{-1} \mu_t}{1_N \Sigma_t^{-1} \mu_t} \quad (3)$$

[13] Notice that we obtain a lean Black-Litterman model from a Markowitz (1952) framework when we set views to be same as priors. In this case, equilibrium returns and the shrunk covariance matrix are the model inputs. Views can induce subjectivity in the allocation process. Using notation in Black and Litterman (1992), in this case confidences Ω may be set proportionally to the variance of the priors $\prod$ ie $\Omega = \tau P \sum P^T$. We constructed views from a sentiment-based index using three of the Baker and Wurgler (2006) factors: Change in Traded Volumes on US Equity Markets, the Volatility Index and SKEW Index from the Chicago Board Options Exchange to develop views as in the classic Black-Litterman model and obtained qualitatively similar results.

To evaluate portfolio return properties, we conduct two kinds of tests. The first test involves holding a strategy from the beginning till the end of the period. The second test involves performing in-sample out-of-time tests to evaluate portfolio performance. As a direct consequence of the former case, W is static and the varying factor is the set of imposed constraints and the estimation of μ and Σ. For these tests, we also incorporate a 1% linear transaction cost model for each portfolio rebalancing.[14]

3.3. Portfolio strategies

Naïve Diversification (1/N)

The naïve strategy involves assigning weights of 1/N to each of the N risky assets in the portfolio. Portfolio optimization and covariance matrix estimation are not required. Naturally, we impose constraints that expected returns are proportional to total risk rather than systematic risk. DeMiguel et al. (2009) show that, out-of-sample, the gain from optimal diversification is more than offset by estimation errors when compared with the naïve 1/N strategy. The naïve strategy offers the simplest form of portfolio diversification and is most intuitive and easy to implement.

Long Only Strategy

For the long only strategy, we impose non-negativity constraints on portfolio weights (no short-selling allowed in this strategy) in optimization problems. All positions are entered into at the beginning of our sample period and exited at the end. In addition to all of the above, our convex system is subjected to the following extra constraints:

$$w: w_i \geq 0$$

$$\sum_{\forall\, i \in N} w_i = 1$$

[14] We make an exception for the Naïve Diversification as positions are constant from beginning to end of period.

Long – Short Strategy

For this strategy, we relax the non-negativity constraints. Short positions or leverage may provide flexibility and may be beneficial in scaling trading strategies. Weight constraints for this strategy change to:

$$w\colon -1 \leq w_i \leq 1\,, \qquad \sum_{\forall\, i \in N} w_i = 1$$

Minimum Variance Strategy

While we do not seek to test strategy outperformance, a wide body of literature (see Kritzman et al., 2010) suggests that minimum variance portfolios formed from optimization often perform better out-of-sample given the difficulty of forecasting expected returns. Behr et al. (2013) find that a constrained minimum-variance strategy outperforms a naïve (1/N) portfolio Sharpe Ratio-wise, in contrast to DeMiguel et al. (2009). In our case, we only aim to compare portfolio strategy performances with and without emissions products. We choose portfolios of risky assets that minimize the volatility of returns, i.e.

$$\min_{W_t} w_t^T \Sigma_t w_t \qquad \sum_{\forall\, i \in N} w_i = 1$$

3.4 Portfolio performance measurement

To assess performance of portfolios that include EUA and/or GEO and NGEO, and portfolios that do not, we conduct in-sample-out-of-time tests to check for differences in portfolio Sharpe Ratios. Consider the dataset to be T days long. In the first step, we rely on a rolling sample of t-q days history with which we initially estimate the portfolio parameters μ, Σ and solve the optimization problem in (2) for the respective strategies to obtain optimal weights W_t. Given the ensuing parameters, we evaluate portfolio performance subsequently on the basis of portfolio return and volatility. Specifically, at time t, W_t becomes the investor's portfolio position for the next t + j days. The algorithm progresses by looking at (t-q) days and evaluating portfolio performance for the next (t + j) days as earlier observations in the data series get dropped one day at a time. The algorithm outcome is a time series of

T-q long in-sample-out-of-time series of portfolio returns, volatility and Sharpe Ratios for each strategy. We compute the in-sample-out-of-time Sharpe ratio for each strategy according to equation (4).

3.5 Performance analysis

We use the series of portfolio returns and Sharpe Ratios described above as the basis of the performance analyses. First, we compute the ith difference between generated Sharpe Ratios for portfolios of the same strategy as for all portfolio pairs (j, k) as:

$$\widehat{\Delta}_i = \widehat{SR_{j_i}} - \widehat{SR_{k_i}}$$

We then conduct a robust test for difference in Sharpe Ratios as outlined in Ledoit and Wolf (2008). We present p-values corresponding to an inverted studentized bootstrap confidence interval for the difference in the Sharpe Ratios. There is an extensive literature suggesting improved inferential accuracy of the studentized bootstrap over 'standard' inference based on asymptotic normality (Hall, 1992) for time series data (Lahiri, 2003). To generate bootstrap data, we generate stationary block bootstraps according to Politis and Romano (1992) and resample blocks of pairs from the observed pairs $(r_{tj}, r_{tk})'$ $t = 1, \dots, T$ with replacement for return pair series generated in the previous section. Stationary bootstrap blocks have a fixed size b≥1. Ledoit and Wolf (2008) further provide an iterative approach for the b selection. We generate 10,000 bootstrap samples and test $H_0\colon \Delta_{SharpeRatio} = 0$ at significance level α applying the decision criteria outlined above.[15]

4. Empirical results

The aim of the tests described in the previous section was to find out if portfolios with EUA, GEO, NGEO or a combination of the emissions products perform better than portfolios without.[16] Figure 5 presents efficient frontiers based on portfolio specifications with (Benchmark + C) and without

[15] Notice that the studentized statistic is approximated as $\zeta\left(\frac{||\widehat{\Delta}-\Delta||}{s(\widehat{\Delta})}\right) \triangleq \zeta\left(\frac{|\widehat{\Delta}^{*}-\Delta|}{s(\widehat{\Delta}^{*})}\right)$, where Δ is the true difference between Sharpe Ratios, $\widehat{\Delta}$ the estimated difference computed from the original data, s the respective standard deviation and $\widehat{\Delta}^{*}$ the Sharpe difference computed from bootstrap data. See Ledoit and Wolf (2008) for further details.
[16] Portfolio comparison definitions are provided in Table 10.

(Benchmark) EUA. The difference is stark. Adding EUA to the other assets results in a significant increase in the level of the efficient frontier. In Figure 6, a portfolio that integrates EUA and GEO (Benchmark + GC) or GEO alone (Benchmark + G) yields an efficient frontier higher than a portfolio of the other asset classes (Benchmark). Figure 7 shows efficient frontiers obtained from portfolio combinations including various combinations of EUA, GEO and NGEO with the benchmark portfolio. As in Figures 5 and 6, including NGEO (Benchmark + N) in Figure 7 significantly raises the efficient frontier of optimal portfolio choices. Notice that the efficient frontiers of portfolios with EUA, GEO and NGEO in Figures 8 to 10 are all steeper than the efficient frontiers of portfolios without NGEO, GEO and EUA in this period. This result is key to the rest of this section: on a risk-adjusted basis, the introduction of EUA, GEO or NGEO causes an upward shift of the efficient frontier of optimal portfolio choices.

4.1 In-sample asset allocation and optimization results

Category one data – Benchmark + C vs. Benchmark

Table 4 presents results from the naïve diversification, CAPM and Minimum Volatility strategies showing optimal weights, expected return, volatility and Sharpe Ratios, all annualized. For each strategy, column (1), Benchmark + C incorporates EUA into the portfolio. Column (2) comprises only the benchmark. As with all in-sample portfolio optimization tests in this paper, we assume positions are entered into at the beginning of the period and exited at the end of the period and incorporate a 1% transaction cost.

In column (1) of the table, we see that integrating EUA into the benchmark portfolio yields a Sharpe Ratio of 0.709 based on an annualized 9.7% return and 13.7% volatility. The benchmark portfolio achieves 0.313 based on a 3.9% annualized return and 12.6% volatility over the same period. Our in-sample CAPM strategy combines elements of the Long and Long-Short strategy by construction. Using CAPM-generated returns, we allow for both long and short positions. It is easy to see that based on input parameters in-sample, the Long and Long-Short strategy can yield similar weights.[17] Integrating EUA in our CAPM specification leads to an annualized Sharpe Ratio of 0.860 in (1) versus 0.350 for the benchmark portfolio. CAPM (1) presents a well-diversified portfolio with a 19.5% EUA portfolio weight.

Similarly, the Minimum Volatility strategy leads to a well-diversified portfolio in both (1) and (2). Conspicuously, however, the EUA weight is only 2.6% as the objective is to minimize volatility instead of the Sharpe Ratio.[18] We find that the Minimum Volatility strategy (1) yields a 0.710 annualized Sharpe Ratio compared to 0.300 for the benchmark portfolio.

Overall, integrating EUA into the benchmark portfolio yields the highest Sharpe Ratio (0.860) in CAPM (1). We notice as well that Naïve (1) and Minimum Volatility (1) are at least twice as large as their respective benchmarks (2).

Category two data – Benchmark + GC, Benchmark + G vs. Benchmark

We now present in-sample portfolio allocation and optimization results for category two data for Naïve Diversification, CAPM and Minimum Volatility Strategies in Table 5. As above, column (1) incorporates EUA and GEO (GC) into the portfolio. Column (2) incorporates only GEO (G) into the benchmark, while (3) comprises only the benchmark. Optimal weights in the Naïve strategy are trivially determined by the number of assets incorporated in the portfolio. We find that Naïve (1) performs best on an adjusted risk-return basis, integrating both EUA and GEO into the benchmark (Sharpe Ratio: 0.472). Naïve (2), adding GEO to the benchmark is second best (0.196) with a Sharpe Ratio more than 100% better than the benchmark (-0.180). CAPM (1) and CAPM (2) results are consistent with those reported under Naïve (1) and Naïve (2) with annualized Sharpe Ratios of at least 3 times larger than the ones of the benchmark. EUA (16.7%) and GEO (17.0%) weights are almost even in CAPM (1) with GEO gaining somewhat in the absence of EUA in CAPM (2) (20.3%). In

[17] In our case, the Long and Long-Short strategies yield almost identical positions. Results are then similar for a 2 % minimum weight strategy.

[18] Table 2 shows EUA as one of the most volatile assets. See Appendix Figure 4 in addition.

both cases, we observe the desirable characteristic of high diversification and performance Sharpe Ratio-wise. Minimum Volatility (1) and (2) similarly perform better on a risk-adjusted basis than the benchmark portfolio. We again observe non-zero weights for all assets in the portfolio. As GEO is highly volatile in this period, we observe a rather low average weight (3.8%) for GEO in (1) and (2).

Category three data – Benchmark + NGC, Benchmark + NC, Benchmark + NG, Benchmark + N vs. Benchmark

For category three data, for each of Naïve, CAPM and Minimum Volatility, column (1) incorporates NGEO, GEO and EUA (NGC) into the benchmark portfolio. Column (2) adds NGEO and EUA (NC) to the benchmark. Column (3) combines GEO and NGEO (NG) with the Benchmark while column (4) integrates only NGEO (N). Column (5) comprises only the benchmark. The results are presented in Table 6. Across all three strategies, columns (1) – (4) report Sharpe Ratios larger than the benchmark (column 5), consistent with previous results. While the inclusion of EUA, GEO and NGEO lead to Sharpe Ratio differentiation across all three strategies, reported Sharpe Ratios are however broadly smaller than in data categories one and two, with lower annualized returns and marginally higher volatility especially when using CAPM. Part of this may be attributed to significant economic headwinds over the category three period that is incorporated into risk-adjusted asset returns and covariances. While EUA, GEO and NGEO are among the most volatile assets in this period, we observe that CAPM (5) reports a higher annualized volatility (30% vs. < 22% for each of (1) – (4)).

In summary, the results of this section present EUA, GEO and NGEO as significant Sharpe Ratio contributors to the diversified benchmark portfolios. Adding EUA, GEO or NGEO are associated with large jumps in Sharpe Ratios over those reported by the benchmark.

4.2. Risk-return properties of EUA, GEO and NGEO

The results of the previous section show that the inclusion of emissions-based products EUA, GEO and NGEO yield a superior in-sample risk-adjusted performance measured in the form of higher annualized Sharpe Ratios for the Naïve Diversification, CAPM and Minimum Volatility strategies. To understand how GEO and NGEO contribute to risk-return assessments, we consider the volatility and return spillover results depicted in Tables 7 - 9.

Table 7 shows volatility and return spillovers that reflect category one portfolios. Spillovers are constructed from an underlying VAR(p) process with p determined by selecting optimal lags using the Akaike Information Criterion. For clarity, the estimates on the diagonal represent own shocks, that is, the amount of shocks a particular asset has on its own subsequent volatility while off-diagonals capture the directional spillovers between each asset pair. First, volatility and return spillovers account for only 2.24% of overall forecast error variance but 28.74% for all returns across assets in the portfolio. The aggregate spillover results show a non-trivial amount of connectedness especially amongst asset returns. We observe, albeit small, that EUA is marginally a net transmitter of volatility (0.09%) but a net receiver of returns (-0.68%) within the network. Demand for EUA is ensured by the need for agents to remain compliant with regulation and will rise with increased output across multiple assets, especially in boom times. This is reflected in, for example, equities (9.78%), REITS (7.84%), commodities (3.79%) and currencies (2.04%), which are the highest transmitters of return to the network in general, and to EUA. Some studies[19] also show that more and more corporations factor carbon-related costs into the cost of doing business and which in turn introduces EUA as a volatility transmitting factor across multiple asset classes. EUA is a marginal net transmitter of volatility in the network (0.09%).

The net volatility spillover rises marginally while return spillovers decline in Table 8 compared to Table 7. The level of volatility connectedness rises within our asset universe while return connectedness reduces. This is naturally the case when assets are subjected to external shocks which

[19] See Alessi et al. (2021) and Ehlers et al. (2022).

illicit different responses within a networked system. We observe GEO to be a net recipient of volatility (-2.42%) and returns to a lower extent (-0.56%) with bonds being the largest transmitter of volatility (2.51%) and returns (0.18%) to GEO. The bond market is generally a key indicator for the health of the economy. While no direction is implied, this result suggests GEO, like most assets is sensitive to shocks to the broader economy. It also signals potential sensitivity to external or exogenous shocks that impact the global economy. Meanwhile, EUA remains a net transmitter of volatility and a marginal return recipient. EUA is not as sensitive to volatility shocks in the bond market (0.01%) but is the second largest return recipient from bonds (0.35%), after currencies (0.42%).

In Table 9, aggregate volatility connectedness of the network is markedly higher (9.58%) for category three portfolios while return connectedness is reported at a non-trivial 27.33%. GEO is still a net volatility (-2.42%) and marginal return (-0.19%) recipient. NGEO itself is a net recipient of volatility (-1.37%) and returns (-3.44%) in the network. The volatility spillover from bonds to GEO is 2.47% in category three and next largest to NGEO (0.42%). Similarly, NGEO, the sector that focuses on AFOLU-related projects in VCMs may also be sensitive to exogenous shocks. In category three, however, EUA is both a net transmitter of volatility and returns to the system. The EUA result is more synonymous to the fact that regulatory compliance to emission abatement rules has forced compliance carbon allowances to become a commodity capable to inducing shocks within a system.

The result of the spillover analysis suggests that from a macro and fundamental perspective, positive shocks within the network will positively impact GEO and NGEO, while large negative asymmetric shocks will have large negative consequences for price discovery and ultimately voluntary carbon projects' viability. Compliance carbon allowances, on the contrary, may be more resilient to exogenous shocks as by definition compliance carbon allowances are backed by the regulator. These results are consistent with Khalfaoui et al. (2022) who find that the spillover connectedness network of the US stock market is very sensitive to market states and that the strength of the effects of climate change-related risks are more pronounced under bust and boom markets.

4.3. Portfolio strategy performance

In this section, we present in-sample-out-of-time performance results of all portfolio allocations across the four discussed strategies. As outlined above, the outperformance tests work as follows. We estimate optimal portfolio weights by solving the portfolio optimization problem in equation (2) using the last 90 days[20] and conduct performance analyses for the next 5, 10 and 20-day holding periods. Return and Sharpe Ratios over the holding periods are calculated at the end of the respective period and the estimation window shifts by one day. Finally, we construct a studentized bootstrap out of the time series of Sharpe Ratios and test for the significance in difference in Sharpe Ratios between the constructed portfolios using the methodology described in Ledoit and Wolf (2008). Table 10 provides definitions of the portfolios used in this section.

Category one data – Benchmark + C vs. Benchmark

Table 11 provides estimated annualized Sharpe Ratios per strategy for the Benchmark + C and Benchmark portfolios respectively from the return bootstraps. *Diff* accounts for the difference in the risk-adjusted performance estimates measured, for which we test for statistical significance for subsequent 5, 10 and 20-day holding periods. We observe that the reported Sharpe Ratios for Naïve, Long, and Minimum Volatility are larger for Benchmark + C than the benchmark portfolio across all holding periods. We observe that this difference is statistically significant at the 99% level. While Benchmark + C does not outperform the benchmark portfolio for 5 and 10-day holding periods in the Long-Short strategy, the result for 20-day holding periods is significant.

Category two data – Benchmark + GC, Benchmark + G vs. Benchmark

Table 12 presents in-sample out-of-time performance results for category two data for the four strategies. We observe that Benchmark + G and Benchmark + GC report larger and statistically significant Sharpe Ratios above the benchmark for the Long, and Long-Short strategy across all holding periods.

[20] We apply a 90-day history to achieve a reasonable amount of degrees of freedom for the GEO, NGEO time series.

As with category one data, the size of Sharpe Ratio differences is proportional to the holding periods. Longer holding periods may also reduce transaction costs even for relatively volatile assets through periods of stress. While the Naïve and Minimum Volatility strategies report larger Sharpe Ratios across almost all holding periods (except Minimum Volatility: Benchmark + GC), the results do not attain conventional significance and hence are unable to reject the null hypothesis of zero Sharpe Ratio difference between the emission-integrated and benchmark portfolios.

Category three data – Benchmark + NGC, Benchmark + NC, Benchmark + NG, Benchmark + N vs. Benchmark

Category three data focuses on integrating NGEO, GEO and EUA in the different possible combinations in the benchmark portfolio in Table 13. All variables have their usual meanings. Contrary to results obtained until now, Sharpe Ratio differentiation in the Naïve strategy favors the benchmark portfolio over Benchmark + N, Benchmark + NG and Benchmark + NGC in multiple holding periods. For example, Benchmark + N reports a Sharpe Ratio of -2.679 vs. -2.055 for the benchmark. These results are consistent with the Minimum Volatility strategy where the benchmark outperforms portfolios with NGEO, GEO and EUA across all holding periods. We do not observe a Sharpe Ratio differentiation in the Long Strategy. Finally, the Long-Short strategy reports Sharpe Ratios for portfolios that include NGEO, GEO and EUA, which are generally significantly larger than the benchmark.

DeMiguel et al. (2009) showed that the promised gain of out-of-sample diversification effectively does not fare better than the 1/N rule. While this empirical result is true for strategy comparisons, in a connected economy, large idiosyncratic shocks may propagate through the network and generate large aggregate fluctuations (Acemoglu, 2015; Zareei, 2019). The 1/N results inadvertently present short-run evidence that passive diversification may be susceptible to exogenous shocks for a highly connected network, which features dominantly in our dataset (Russia-Ukraine war, Covid pandemic, inflationary pressures) in the absence of active optimization. To make sense of the results in this section, we graphically present the rolling optimal weights for EUA for category one in Figure 8; GEO + EUA and GEO for category two in Figure 9; and NGEO + GEO + EUA, NGEO + GEO and NGEO for category three in Figure 10.

In Figure 8, we observe the rolling optimal, yet volatile, EUA weights in the Long and Long-Short strategies are identical with a key difference being periodic short-selling positions accommodated in the Long-Short strategy (Long strategy falls to zero). We observe that rolling optimal weights range between -0.6 and 0.3 in the first half of 2020 coinciding with the onset of the COVID-19 pandemic. Optimal long EUA positions recover, and assume short positions only briefly in the second half of 2021. Using estimation histories of 90 days, rolling optimal long positions in EUA are interrupted at the onset of the Russia-Ukraine war with now short positions reflecting market pessimism and concerns in Europe about energy security and inflation.[21]

Figure 9 amplifies the GEO sensitivity to exogenous shocks better. “Optimal weights: GEO + EUA” sums the rolling optimal weights for GEO and EUA across the four strategies while “Optimal weights: GEO” shows only GEO weights. Focusing on the Long-Short strategy, it becomes clear that GEO positions generally drop off significantly at the onset of the war in Ukraine and do not truly recover, while EUA weights are responsible for the perturbations and weight volatility observed in “Optimal weights: GEO + EUA”. GEO weights in the Long Strategy drops off sharply to zero while the Minimum Volatility Strategy also reduces significantly to almost zero for all strategies. In the optimal flight away from GEO, GEO allocation in the Naïve Strategy remains unchanged. Figure 10 is entirely consistent with Figure 9. In fact, NGEO and GEO dominate aggregate short-selling positions in Figure 10, compared to EUA at the onset of the war.

The key takeaways from this section are as follows. While GEO and NGEO show evidence of return and ultimately Sharpe Ratio differentiation in portfolios, they also show a high sensitivity to exogenous shocks. EUA is fundamentally different. While the economy may observe contractions

[21] Optimization input parameters are based on t-90 days, which is then incorporated into subsequent positions taken for holding periods ahead.

and economic security become threatened, compliance carbon allowances may be more resilient as regulated emissions have increasingly become a balance sheet item. In the fight against climate change, each emission reduction counts. Fundamentally, this result may also be interpreted as follows: voluntary abatements become less urgent when having to deal with large negative exogenous shocks. Appendix Table 1 provides summary statistics on the optimal rolling weights for EUA, GEO and NGEO for the in-sample out-of-time tests.

4.4. Is it worth the while incorporating carbon offsets into an investor´s portfolio allocation?

The results of this study provide empirical reasons for integrating compliance allowances and voluntary carbon offsets in investment portfolios. One caveat regarding our results is that GEO and NGEO were launched only in 2021. Could the results therefore be biased due to potentially early interest from sustainable investors and corporate net-zero pledges? We reduce such potential bias by constructing benchmark portfolios as controls and backing out risk-adjusted expected market returns in all three categories for the same period for which EUA, GEO and NGEO exist to observe how the possible different combinations perform. If voluntary carbon offsets do not significantly contribute to the Sharpe Ratio, then a portfolio without them should perform similarly or better using the same category of reference prices.

The performance of portfolios with voluntary carbon offsets based on return-covariance properties may only be one side of the story, which naturally raises the question of whether its risk-reward properties may persist? Fundamentally, while the demand of compliance carbon is necessary for firms to remain regulatorily compliant, demand for voluntary carbon offsets is implicitly secured through corporate net zero pledges, increased awareness of households and policymakers and, crucially, the marketplaces for other, traditional asset classes. Sustainability-conscious investing, coupled with the growing appetite for clean technologies – usually a by-product of voluntary and compliance carbon pricing – provide fundamental reasons beyond the moral argument for integrating compliance carbon allowances and voluntary carbon offsets into portfolios. Nordhaus (2011) describes this as *price fundamentalism*, that is, under limited conditions, a necessary and sufficient condition for an appropriate innovational environment is a universal, credible and durable price on emissions which balances the marginal damages from emissions against abatement costs. A such, while voluntary carbon offset price histories are short and the market itself shows reliability issues and exogenous shock sensitivity, we expect the risk-reward properties documented in this study to remain robust.

5. Conclusions

In this paper, we study in-sample and in-sample-out-of-time performances of portfolios that integrate emissions products, proxied by compliance allowances and voluntary carbon offsets, versus those that do not in using the naïve, CAPM and minimum volatility strategies. Based on portfolio performance measured by the Sharpe Ratio, portfolios that integrate voluntary (GEO and NGEO) and compliance carbon allowances (EUA) consistently outperform portfolios that exclude them in-sample. We observe that the efficient frontier of optimal portfolio choice is shifted upwards upon introducing EUA, GEO or NGEO. Hence, the portfolio Sharpe component attributable to EUA, GEO and NGEO must be significant. The driving factors behind the portfolio diversification benefits of GEO and NGEO include i) low correlation with other asset classes due to unique price discovery in voluntary carbon markets; and ii) that voluntary carbon offsets are volatility and return spillover recipients in most of the portfolios we study in a manner that is consistent from a macro perspective.

To gauge outperformance, we measure in-sample-out-of-time Sharpe Ratios of the naïve diversification, long, long-short and minimum variance strategies and test the difference in Sharpe Ratios between portfolios with integrated emissions products and those without. We find that a benchmark portfolio that includes EUA consistently outperforms the benchmark. Similarly, a basket of portfolios that integrates GEO or GEO and EUA broadly outperforms the benchmark. However, the benchmark portfolio outperforms a benchmark portfolio that includes different combinations of NGEO, GEO and EUA for Naïve Diversification and Minimum Volatility, while the reverse is true for the long-short strategy we study in-sample out-of-time. We show in our volatility and return spill-over analyses, as well as in our in-sample out-of-time tests that the results especially show voluntary carbon markets' sensitivity to exogenous shocks. In addition, the global de-carbonization trend provides a fundamental argument for integrating voluntary carbon offsets into portfolios. In many ways, however, our results also provide empirical evidence of how voluntary abatements become secondary priority in the face of exogenous shocks – a result that flouts against the well-accepted notion of 'all reductions count' in the fight against climate change.

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Figures

Figure 1: Price time series of all assets

The figure below shows price development of all assets indexed against the respective initial price. The data covers the period 07/10/2017 – 13/10/2022 with the exception of GEO (inception: 01/03/2021) and NGEO (inception: 04/08/2021). Table 1 provides a description of the assets listed and sources from whence obtained.

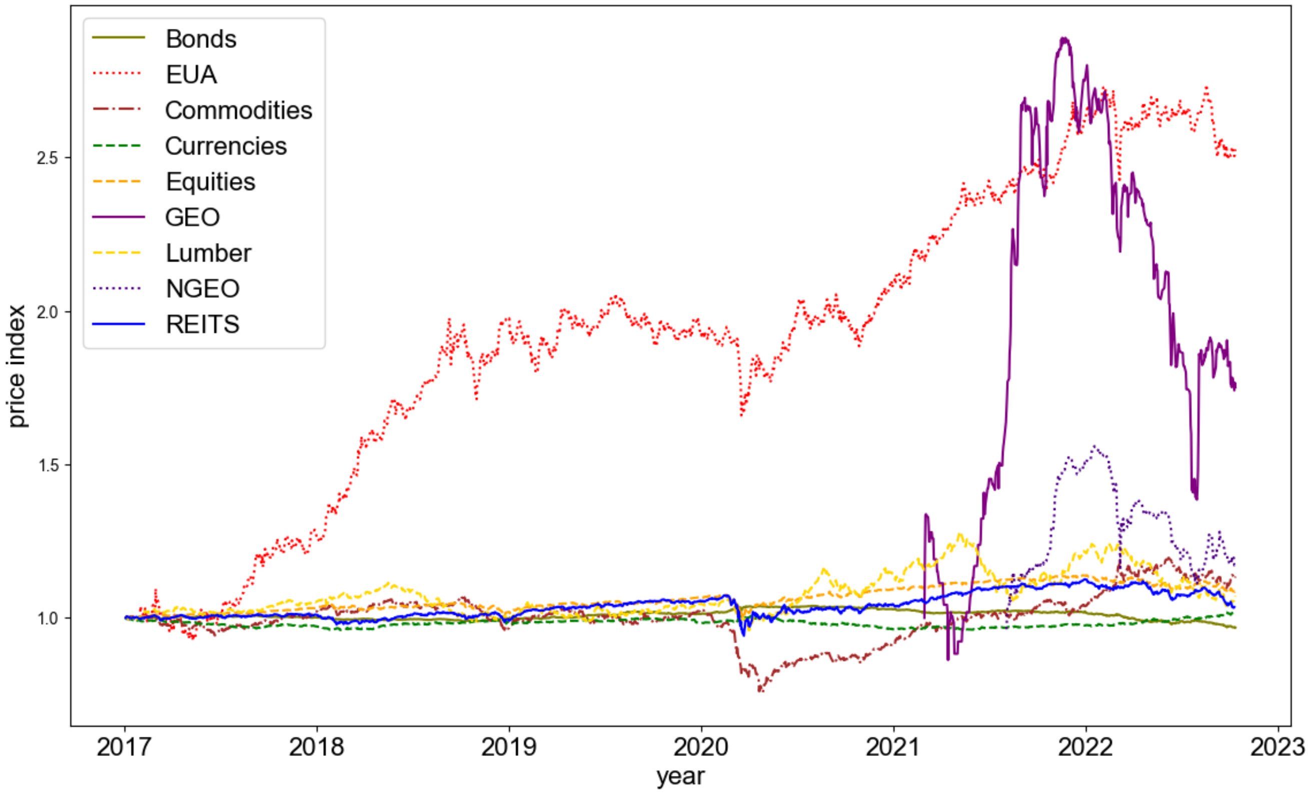


Figure 2: Category one data return correlations (p-values > 0.05 masked)
The figure below is a correlation matrix of daily asset returns for category one data (07/10/2017 – 13/10/2022). Statistically significant correlation pairs are marked by the color scale. Masked (blank) fields refer to asset pair correlations that miss conventional statistical significance (p-value > 0.05).

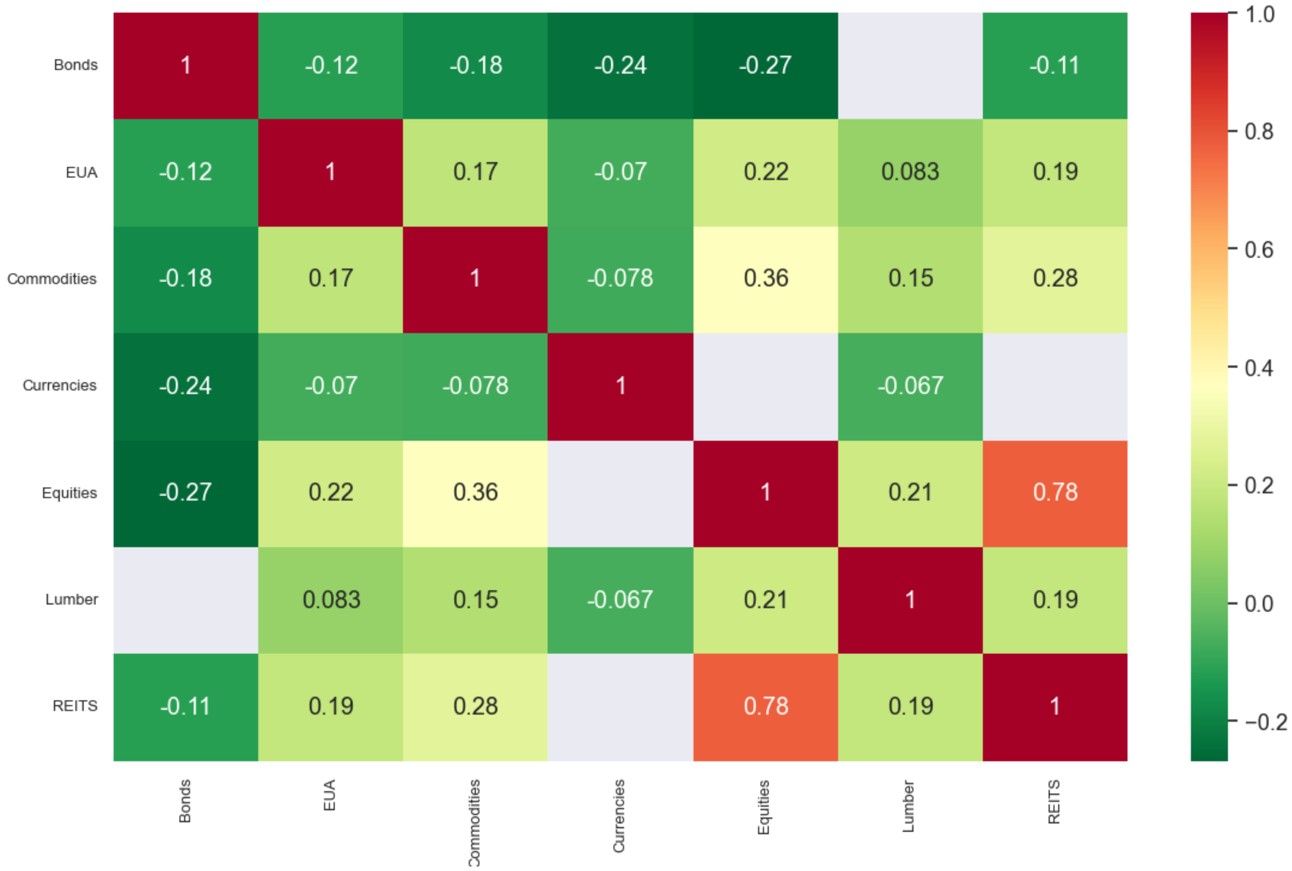


Figure 3: Category two data return correlations (p-values > 0.05 masked)
The figure below is a correlation matrix of daily asset returns for category two data (01/03/2021 – 13/10/2022). Statistically significant correlation pairs are marked by the color scale. Masked (blank) fields refer to asset pair correlations that miss conventional statistical significance (p-value > 0.05).

Figure 4: Category three data return correlations (p-values > 0.05 masked)
The figure below is a correlation matrix of asset returns for category three data (04/08/2021 – 13/10/2022). Statistically significant correlation pairs are marked by the color scale. Masked (blank) fields refer to asset pair correlations that miss conventional statistical significance (p-value > 0.05).

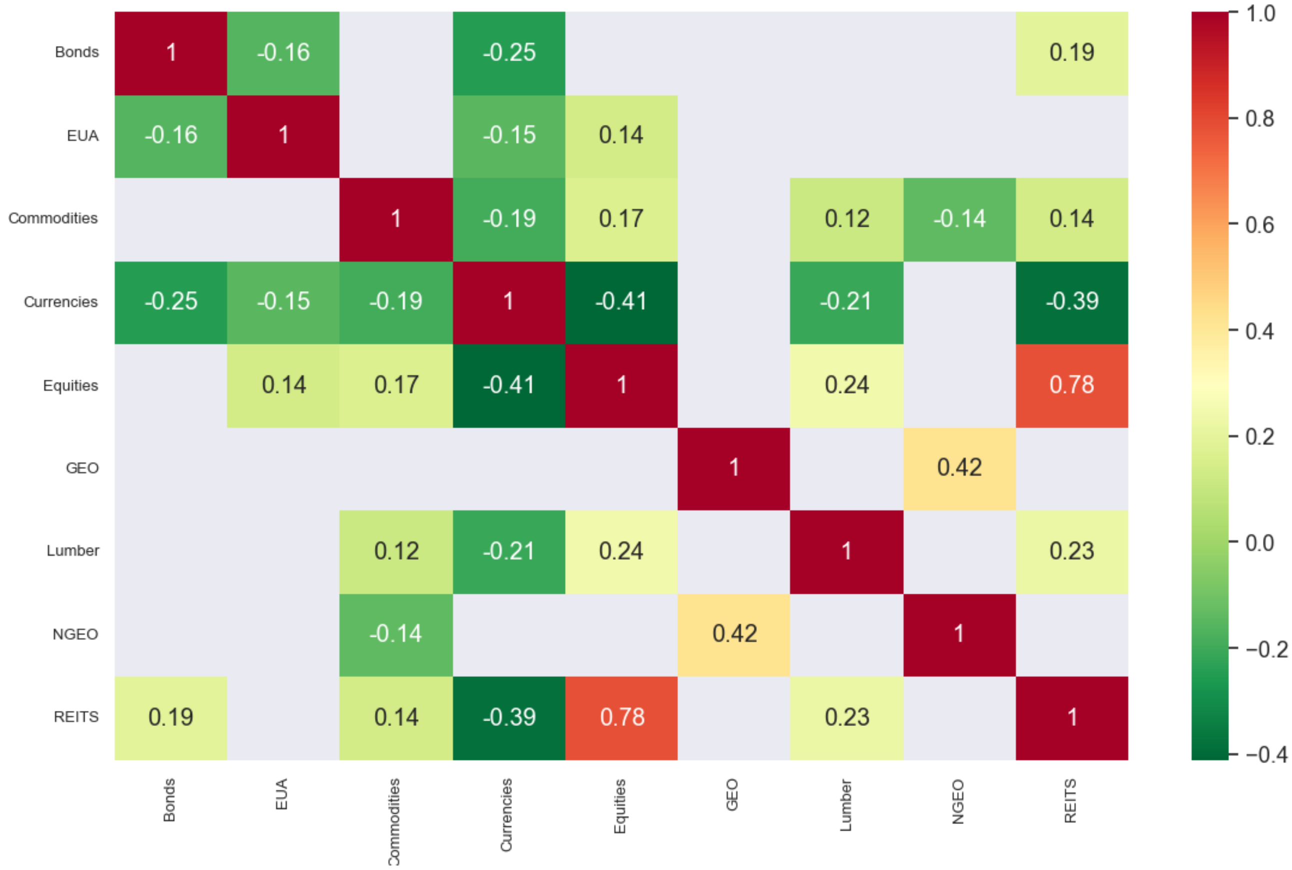


Figure 5: Efficient frontier comparison of portfolios constructed out of category one data

The figure below provides an efficient frontier comparison of the benchmark asset portfolio ("Benchmark") and a benchmark portfolio that integrates EUA ("Benchmark + C"). Annualized expected portfolio returns (%) are displayed on the vertical axis while annualized portfolio volatility (%) is shown on the horizontal axis. Expected returns are backed out of a CAPM-model with the an equally-weighted benchmark as the reference market. The CAPM-returns then serve as the respective market risk-adjusted expected returns. The covariance matrix is estimated using the covariance shinkage approach outlined in Ledoit-Wolf (2004).

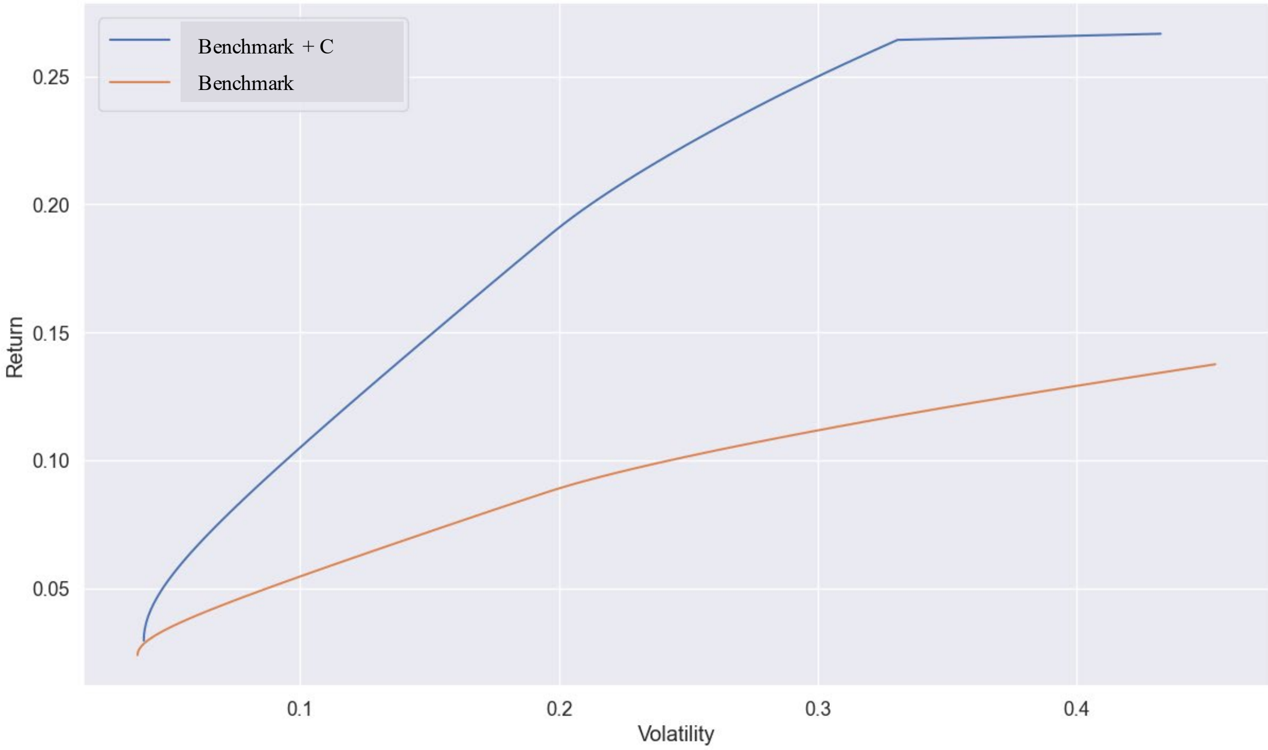


Figure 6: Efficient frontier comparisons for portfolios constructed out of category two data
The figure below provides an efficient frontier comparison of the benchmark portfolio ("Benchmark") and a benchmark portfolio that integrates either GEO and EUA ("Benchmark + GC") or GEO alone ("Benchmark + G"). Annualized expected portfolio returns (%) are displayed on the vertical axis while annualized portfolio volatility (%) is shown on the horizontal axis. Expected returns are backed out of a CAPM-model with the an equally-weighted benchmark as the reference market. The CAPM-returns then serve as the respective market risk-adjusted expected returns. The covariance matrix is estimated using the covariance shinkage approach outlined in Ledoit-Wolf (2004).

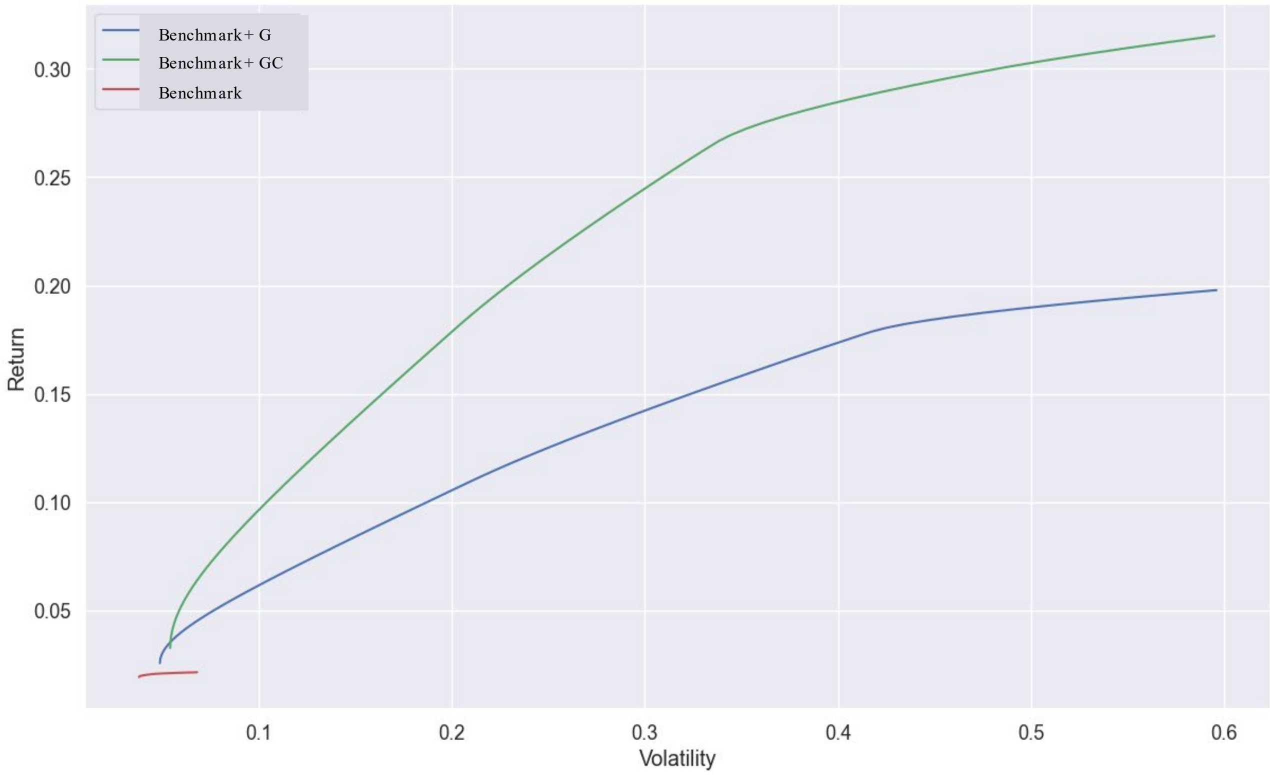


Figure 7: Efficient frontier comparisons for portfolios constructed out of category three data
The figure below provides an efficient frontier comparison of the benchmark portfolio ("Benchmark") and Benchmark portfolio that integrates either NGEO (N), GEO (G), and/or EUA (C). Annualized expected portfolio returns (%) are displayed on the vertical axis while annualized portfolio volatility (%) is shown on the horizontal axis. Expected returns are backed out of a CAPM-model with the an equally-weighted benchmark as the reference market. The CAPM-returns then serve as the respective market risk-adjusted expected returns. The covariance matrix is estimated using the covariance shinkage approach outlined in Ledoit-Wolf (2004).

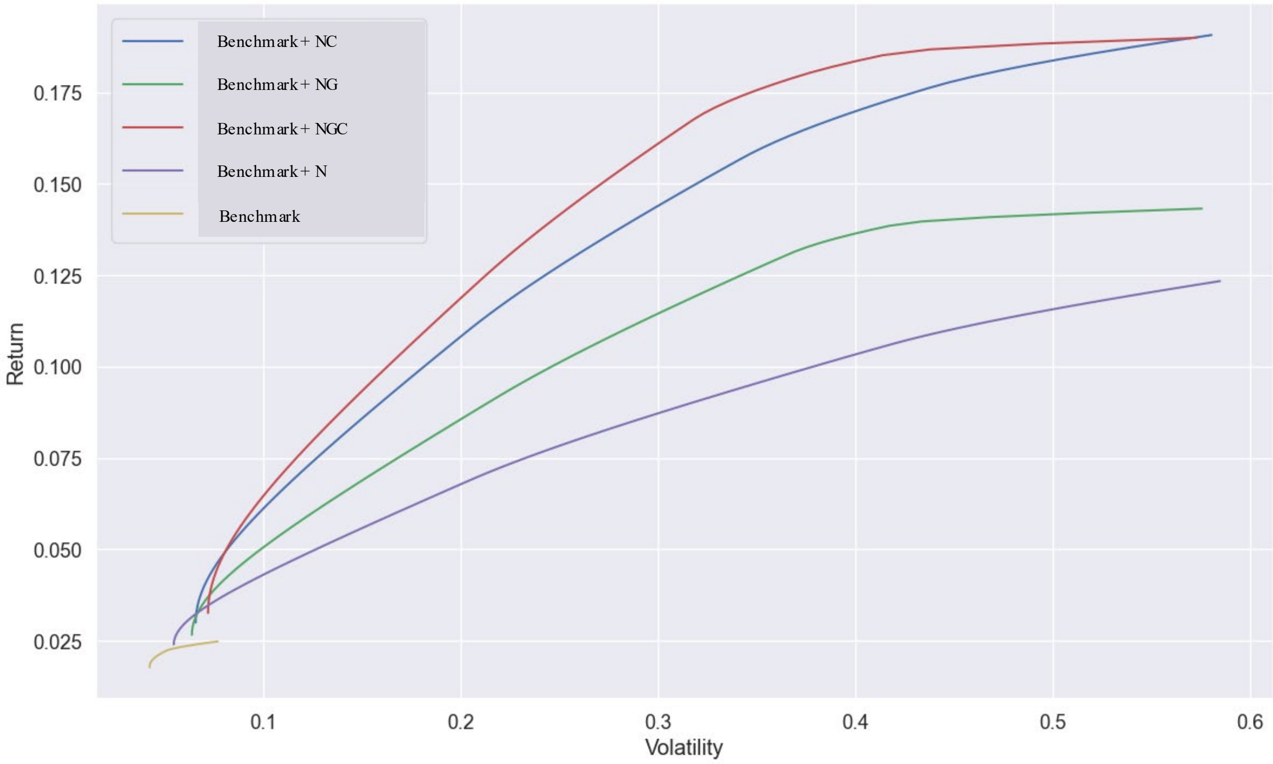


Figure 8: Rolling optimal weights per strategy for EUA using category one data

The figure below shows the rolling optimal weights per strategy for compliance carbon (EUA) using category one data. The vertical axis shows the Weight (./100) while the horizontal axis displays the Date. Optimal weights are determined as follows: Consider the dataset to be T days long. We rely on a rolling sample of t-90 days history with which we initially estimate portfolio parameters μ, Σ and solve the optimization problem for the respective strategies to obtain optimal weights *w* at time *t* for the respective 5-, 10- and 20-day holding periods. Expected returns are backed out of a CAPM-model with the an equally-weighted benchmark as the reference market. The CAPM-returns then serve as the respective market risk-adjusted expected returns. The covariance matrix is estimated using the covariance shinkage approach outlined in Ledoit-Wolf (2004). This scheme is applied to the Long, Long-Short and Minimm Volatility Strategies. Weight allocation in Naïve Diversification is in this case trivial.

Figure 9: Rolling optimal weights per strategy for GEO + EUA and GEO using category two data

The figure below shows the rolling optimal weights per strategy for GEO + EUA, and GEO using category two data. The vertical axis shows the Weight (./100) while the horizontal axis displays the Date. Optimal weights are determined as follows: Consider the dataset to be T days long. We rely on a rolling sample of t-90 days history with which we initially estimate portfolio parameters μ, Σ and solve the optimization problem for the respective strategies to obtain optimal weights w at time t for the respective 5-, 10- and 20-day holding periods. Expected returns are backed out of a CAPM-model with the an equally-weighted benchmark as the reference market. The CAPM-returns then serve as the respective market risk-adjusted expected returns. The covariance matrix is estimated using the covariance shinkage approach outlined in Ledoit-Wolf (2004). This scheme is applied to the Long, Long-Short and Minimm Volatility Strategies. Weight allocation in Naïve Diversification is in this case trivial.

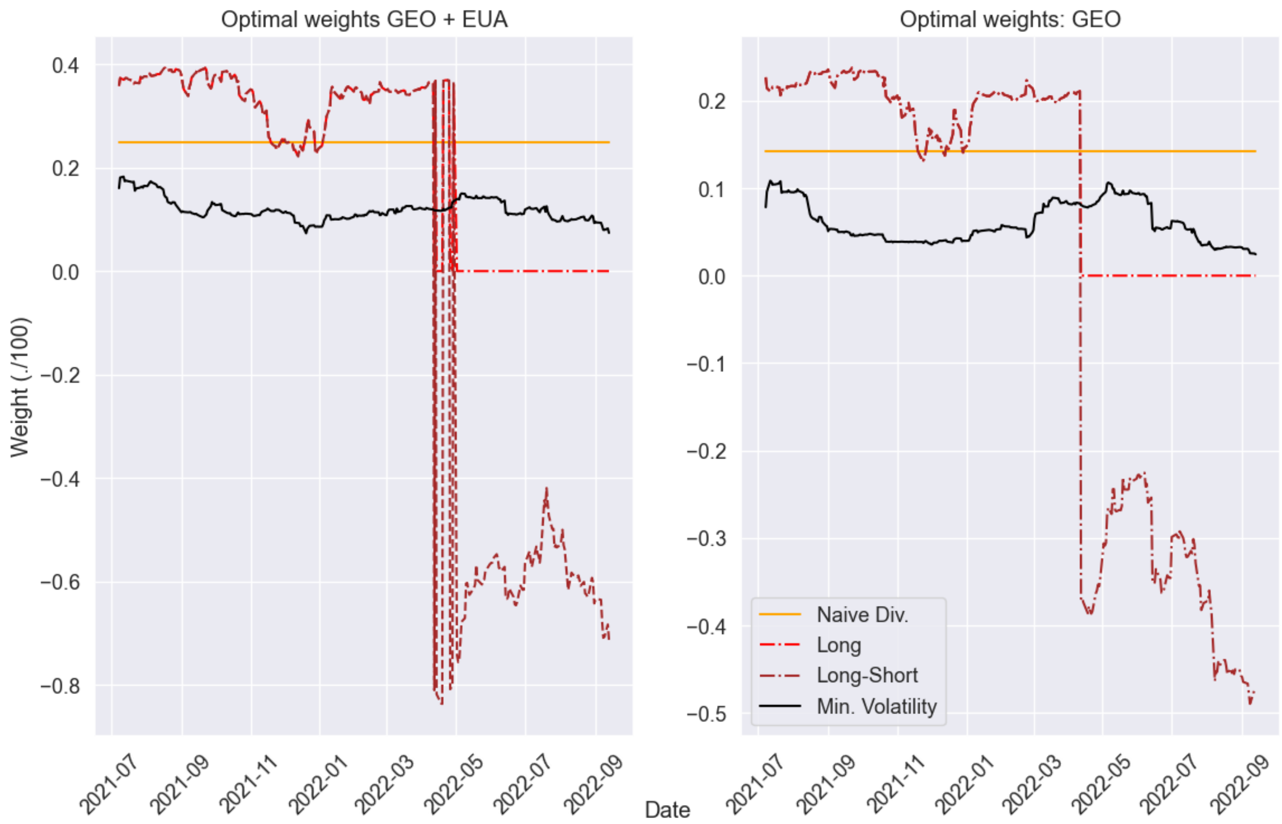
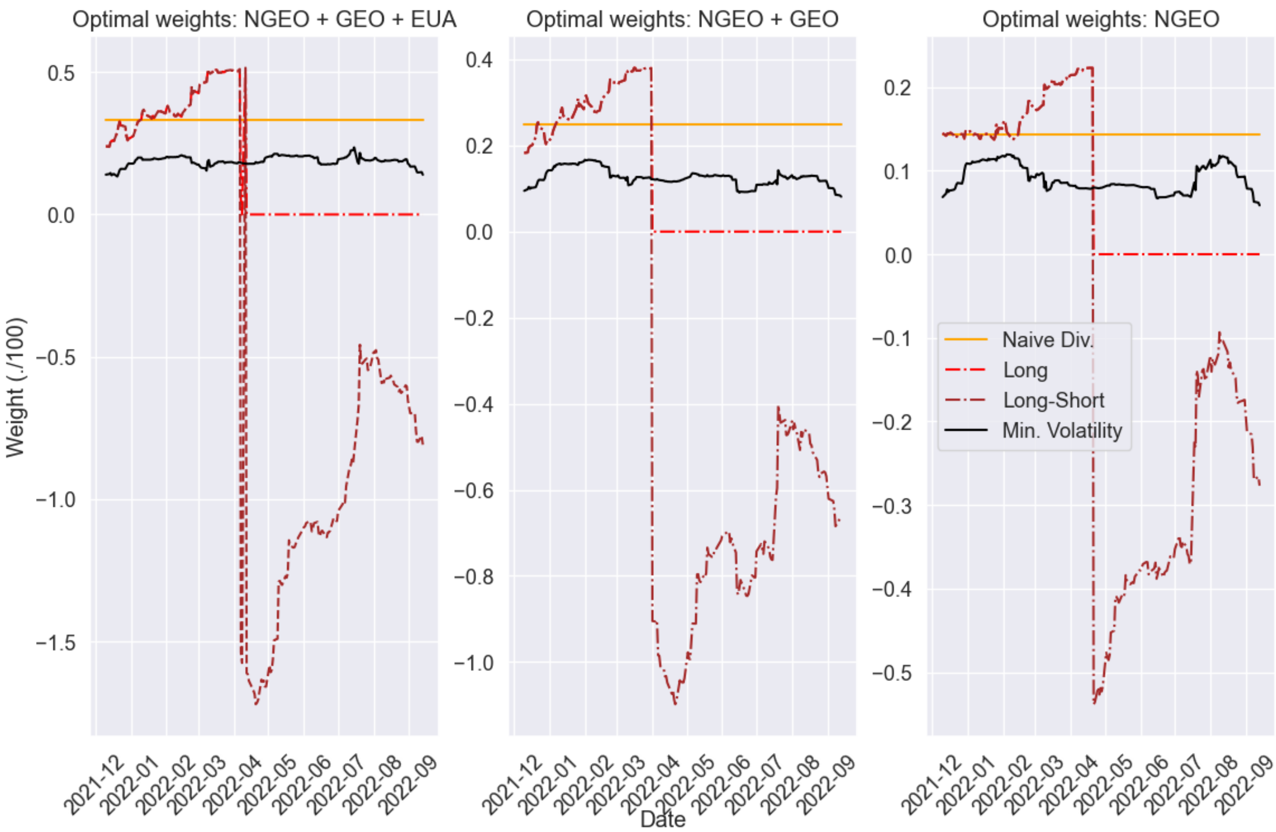


Figure 10: Rolling optimal weights per strategy for NGEO + GEO + EUA, NGEO + GEO, and NGEO using category three data

The figure below shows the rolling optimal weights per strategy for NGEO + GEO + EUA, NGEO + GEO and NGEO using category three data. The vertical axis shows the Weight (./100) while the horizontal axis displays the Date. Optimal weights are determined as follows: Consider the dataset to be T days long. We rely on a rolling sample of t-90 days history with which we initially estimate portfolio parameters μ, Σ and solve the optimization problem for the respective strategies to obtain optimal weights w at time t for the respective 5-, 10- and 20-day holding periods. Expected returns are backed out of a CAPM-model with the an equally-weighted benchmark as the reference market. The CAPM-returns then serve as the respective market risk-adjusted expected returns. The covariance matrix is estimated using the covariance shinkage approach outlined in Ledoit-Wolf (2004). This scheme is applied to the Long, Long-Short and Minimm Volatility Strategies. Weight allocation in Naïve Diversification is in this case trivial.



Tables

Table 1: Asset definitions and data sources

The table lists the names of assets, description and data sources used

Asset	Description	Data Source
Bonds	**The iShares U.S. Treasury Bond ETF**: seeks to track the investment results of an index composed of U.S. Treasury bonds (ICE US Treasury Core Bond Index). The Index is market value weighted, and is designed to include U.S. dollar denominated, fixed rate securities with minimum term to maturity greater than one year and less than or equal to thirty years.	BarChart
EUA	**European Emissions Allowances of the European Emissions Trading Scheme (EU ETS)**: is a cornerstone of the EU's policy to combat climate change and its key tool for reducing greenhouse gas emissions cost-effectively. It is the world's first major carbon market and remains the biggest one as of December 2021.	BarChart
Commodities	**S&P GSCI Commodity-Indexed Ishares ETF:** The iShares S&P GSCI Commodity-Indexed Trust (the 'Trust') seeks to track the results of a fully collateralized investment in futures contracts on the S&P GSCI(R) Total Return Index composed of a diversified group of commodities futures.	BarChart
Currencies	**The WisdomTree Bloomberg U.S. Dollar Bullish Fund ETF:** seeks to provide total returns, before expenses, that exceed the performance of the Bloomberg Dollar Total Return Index	BarChart
Equities	**S&P 500 SPDR ETF** aims to track the Standard & Poor's (S&P) 500 Index	BarChart
GEO	**Global Emissions Offsets**: is a physically settled contract that allows for delivery of CORSIA eligible voluntary carbon offset credits from three registries: Verified Carbon Standard (VCS), American Carbon Registry (ACR), and Climate Action Reserve (CAR).	BarChart
Lumber	Random Length Lumber month-ahead futures	DataStream
NGEO	**Nature-based Global Emissions Offsets**: offer firms a simple way to meet emissions-reduction targets using high-quality, nature-based offsets sourced exclusively from agriculture, forestry, and other land use (AFOLU) projects	BarChart
REITS	**Vanguard Real Estate Index Fund ETF**: tracks the return of the MSCI US Investable Market Real Estate 25/50 Index	BarChart

Table 2: Summary statistics for data category one, category two and category three data

This table shows daily price return descriptive statistics for all datasets used in the empirical analyses. Panel A comprises 6 assets and European Emissions Allowances (EUA) from 07/10/2017 to 13/10/2022. Panel B comprises all assets listed in Panel A but include GEO from 01/03/2021 to 13/10/2022. Panel C includes all assets listed in Panel B but include N-GEO from 04/08/2021 to 13/10/2022. Summary statistics include the number of observations (N), expected returns E[μ] and the Ledoit-Wolf standard deviation LW[σ] backed out of the diagonal of the Ledoit-Wolf shrunk covariance matrix. $E[\mu] = R_f + \beta_i(ER_m - R_f)$: where R_f is the risk-free rate set to 2%, β_i measures the relative risk of asset i compared to the market and ER_m is the expected return on market based on an equally-weighted market benchmark constructed from all assets in the portfolio

	N	E[μ]	LW [σ]
Asset	**Panel A (Category one): Oct 2017 - Oct 2022**		
Bonds	1,435	0.013	0.060
EUA	1,435	0.267	0.454
Commodities	1,435	0.124	0.236
Currencies	1,435	0.018	0.087
Equities	1,435	0.132	0.200
Lumber	1,435	0.265	0.454
REITS	1,435	0.142	0.227
	Panel B (Category two): March 2021 - Oct 2022		
Bonds	408	0.018	0.095
EUA	408	0.224	0.495
Commodities	408	0.086	0.283
Currencies	408	0.004	0.098
Equities	408	0.095	0.201
GEO	408	0.245	0.561
Lumber	408	0.315	0.596
REITS	408	0.090	0.205
	Panel C (Category three): Aug 2021 - Oct 2022		
Bonds	299	0.019	0.129
EUA	299	0.146	0.534
Commodities	299	0.050	0.309
Currencies	299	0.010	0.129
Equities	299	0.070	0.230
GEO	299	0.198	0.579
Lumber	299	0.183	0.582
NGEO	299	0.149	0.490
REITS	299	0.066	0.231

Table 3: Estimated input variance-covariance matrix for in-sample portfolio optimization

The table presents Ledoit and Wolf (2004) shrunk covariance matrix estimates of the asset universe for categories one, two and three. These estimates form the covariance matrices for the respective in-sample asset allocation and optimization analysis of the data sample used across three time series categories. The operational shrinkage estimator of the covariance matrix Σ is given by: $\hat{\Sigma}_{shrink} = \widehat{\delta^*}F + (1 - \widehat{\delta^*})S$ where $0 \leq \delta^* < 1$ is a shrinkage constant that minimizes the expected distance between the shrinkage estimator and the true covariance matrix. We apply a constant variance shrinkage, i.e., target $\delta^* = \mu(\sigma_{11}^2, \dots, \sigma_{jj}^2)$ is the mean of asset variances on the diagonal and zero elsewhere. Leading diagonal reports asset variances while off-diagonals are covariance pair estimates.

Asset	Bonds	EUA	Commodities	Currencies	Equities	Lumber	REITS
				Panel A			
Bonds	0.004	-0.003	-0.002	-0.001	-0.003	-0.001	-0.001
EUA	-0.003	0.207	0.018	-0.003	0.019	0.017	0.018
Commodities	-0.002	0.018	0.056	-0.002	0.017	0.015	0.014
Currencies	-0.001	-0.003	-0.002	0.008	-0.001	-0.002	0.000
Equities	-0.003	0.019	0.017	-0.001	0.040	0.018	0.034
Lumber	-0.001	0.017	0.015	-0.002	0.018	0.206	0.019
REITS	-0.001	0.018	0.014	0.000	0.034	0.019	0.052

Asset	Bonds	EUA	Commodities	Currencies	Equities	GEO	Lumber	REITS
				Panel B				
Bonds	0.009	-0.004	-0.001	-0.001	0.000	-0.003	0.001	0.002
EUA	-0.004	0.245	0.001	-0.005	0.013	0.018	0.017	0.010
Commodities	-0.001	0.001	0.080	-0.003	0.009	-0.012	0.018	0.007
Currencies	-0.001	-0.005	-0.003	0.010	-0.005	-0.001	-0.007	-0.005
Equities	0.000	0.013	0.009	-0.005	0.040	0.005	0.023	0.028
GEO	-0.003	0.018	-0.012	-0.001	0.005	0.315	-0.001	0.003
Lumber	0.001	0.017	0.018	-0.007	0.023	-0.001	0.355	0.018
REITS	0.002	0.010	0.007	-0.005	0.028	0.003	0.018	0.042

Asset	Bonds	EUA	Commodities	Currencies	Equities	GEO	Lumber	NGEO	REITS
					Panel C				
Bonds	0.017	-0.005	-0.001	-0.001	0.000	-0.003	0.001	0.000	0.003
EUA	-0.005	0.285	-0.004	-0.005	0.014	0.018	0.027	0.010	0.012
Commodities	-0.001	-0.004	0.096	-0.004	0.010	-0.011	0.019	-0.019	0.008
Currencies	-0.001	-0.005	-0.004	0.017	-0.006	0.000	-0.008	-0.002	-0.005
Equities	0.000	0.014	0.010	-0.006	0.053	0.009	0.028	0.005	0.032
GEO	-0.003	0.018	-0.011	0.000	0.009	0.335	0.008	0.111	0.008
Lumber	0.001	0.027	0.019	-0.008	0.028	0.008	0.339	0.006	0.026
NGEO	0.000	0.010	-0.019	-0.002	0.005	0.111	0.006	0.232	0.000
REITS	0.003	0.012	0.008	-0.005	0.032	0.008	0.026	0.000	0.053

Table 4: Asset allocation and portfolio optimization results for category one data

Table 4 presents optimization results for the Naïve Diversification, CAPM and Minimum Volatility strategies for category one data from 07/10/2017 – 13/10/2022 showing annualized expected returns, volatility and Sharpe Ratios. Expected returns are backed out of a CAPM model using the asset universe as a market benchmark. Asset covariances are estimated using the Ledoit-Wolf (2004) shrunk covariance approach. For all strategies column (1) is a Benchmark portfolio that incorporates EUA. (2) is the Benchmark portfolio. The Naïve Diversification strategy applies a 1/N weighting to all assets. CAPM and Minimum Volatility uniquely permit short-selling but limits optimal positions to take values between +/-1. The CAPM strategy objective is to maximize the portfolio Sharpe Ratio. The Minimum Volatility strategy minimizes portfolio volatility. It is assumed that positions are held from beginning of period to end-of-period. A very simple transaction cost model sums all the weight changes multiplied by 10 bps, simulating fixed percentage broker commission costs.

	Naive Diversification (1/N)		CAPM		Minimum Volatility	
Asset	(1)	(2)	(1)	(2)	(1)	(2)
Bonds	0.143	0.167	0.023	0.029	0.427	0.445
EUA	0.143		0.195		0.026	
Commodities	0.143	0.167	0.181	0.207	0.136	0.139
Currencies	0.143	0.167	0.033	0.128	0.180	0.171
Equities	0.143	0.167	0.167	0.192	0.143	0.167
Lumber	0.143	0.167	0.196	0.217	0.024	0.024
REITS	0.143	0.167	0.205	0.227	0.065	0.055
Return	0.097	0.039	0.178	0.076	0.067	0.039
Volatility	0.137	0.126	0.185	0.163	0.066	0.065
SR	0.709	0.313	0.860	0.350	0.710	0.300

Table 5: Asset allocation and portfolio optimization results for category two data

Table 5 presents optimization results for the Naïve Diversification, CAPM and Minimum Volatility strategies for category one data from 01/03/2021 – 13/10/2022 showing annualized expected returns, volatility and Sharpe Ratios. Expected risk-adjusted returns are backed out of a CAPM model using the asset universe as a market benchmark. Asset covariances are estimated using the Ledoit-Wolf (2004) shrunk covariance approach. For all strategies column (1) is a Benchmark portfolio that incorporates GEO and EUA. (2) incorporates GEO alone into the benchmark portfolio. (3) is the Benchmark portfolio. The Naïve Diversification strategy applies a 1/N weighting to all assets. CAPM and Minimum Volatility uniquely permit short-selling but limits optimal positions to take values between +/-1. The CAPM strategy objective is to maximize the portfolio Sharpe Ratio. The Minimum Volatility strategy minimizes portfolio volatility. It is assumed that positions are held from beginning of period to end-of-period. A very simple transaction cost model sums all the weight changes multiplied by 10 bps, simulating fixed percentage broker commission costs.

	Naive Diversification (1/N)			CAPM			Minimum Volatility		
Asset	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Bonds	0.125	0.143	0.167	0.019	0.022	1.000	0.181	0.150	0.167
EUA	0.125			0.167			0.040		
Commodities	0.125	0.143	0.167	0.152	0.188	-0.239	0.125	0.130	0.111
Currencies	0.125	0.143	0.167	0.019	0.012	1.000	0.351	0.382	0.397
Equities	0.125	0.143	0.167	0.139	0.169	-0.049	0.125	0.143	0.167
GEO	0.125	0.143		0.170	0.203		0.038	0.038	
Lumber	0.125	0.143	0.167	0.175	0.207	-0.368	0.015	0.014	0.010
REITS	0.125	0.143	0.167	0.158	0.199	-0.343	0.125	0.143	0.148
Return	0.069	0.028	-0.020	0.175	0.106	0.041	0.062	0.042	0.016
Volatility	0.145	0.143	0.139	0.195	0.202	0.293	0.072	0.072	0.066
SR	0.472	0.196	-0.180	0.790	0.430	0.070	0.580	0.300	-0.060

Table 6: Asset allocation and portfolio optimization results for category three data

Table 6 presents optimization results for the Naïve Diversification, CAPM and Minimum Volatility strategies for category one data from 04/08/2021 – 13/10/2022 showing annualized expected returns, volatility and Sharpe Ratios. Expected risk-adjusted returns are backed out of a CAPM model using the asset universe as a market benchmark. Asset covariances are estimated using the Ledoit-Wolf (2004) shrunk covariance approach. For all strategies column (1) is a Benchmark portfolio that incorporates NGEO, GEO and EUA. (2) combines EUA and NGEO with the Benchmark portfolio. (3) is Benchmark portfolio that includes GEO and NGEO. (4) only adds the NGEO to the Benchmark portfolio. (5) is the Benchmark portfolio. The Naïve Diversification strategy applies a 1/N weighting to all assets. CAPM and Minimum Volatility uniquely permit short-selling but limits optimal positions to take values between +/-1. The CAPM strategy objective is to maximize the portfolio Sharpe Ratio. The Minimum Volatility strategy minimizes portfolio volatility. It is assumed that positions are held from beginning of period to end-of-period. A very simple transaction cost model sums all the weight changes multiplied by 10 bps, simulating fixed percentage broker commission costs.

	Naive Diversification (1/N)					CAPM					Minimum Volatility				
Asset	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
Bonds	0.111	0.125	0.125	0.143	0.167	0.011	0.011	0.022	0.054	1.000	0.210	0.179	0.186	0.143	0.167
EUA	0.111	0.125				0.145	0.170				0.043	0.039			
Commodi-ties	0.111	0.125	0.125	0.143	0.167	0.127	0.153	0.153	0.181	-0.235	0.111	0.125	0.125	0.121	0.102
Currencies	0.111	0.125	0.125	0.143	0.167	0.011	0.011	0.009	0.020	1.000	0.323	0.334	0.353	0.383	0.435
Equities	0.111	0.125	0.125	0.143	0.167	0.125	0.146	0.143	0.168	-0.049	0.111	0.125	0.125	0.142	0.157
GEO	0.111		0.125			0.152		0.173			0.019		0.018		
Lumber	0.111	0.125	0.125	0.143	0.167	0.153	0.178	0.173	0.197	-0.364	0.017	0.012	0.015	0.010	0.010
NGEO	0.111	0.125	0.125	0.143		0.145	0.170	0.168	0.192		0.054	0.061	0.054	0.058	
REITS	0.111	0.125	0.125	0.143	0.167	0.132	0.161	0.160	0.189	-0.352	0.111	0.125	0.125	0.143	0.128
Return	0.026	0.024	0.011	0.006	-0.045	0.122	0.108	0.089	0.065	0.078	0.048	0.046	0.038	0.033	0.010
Volatility	0.157	0.146	0.156	0.141	0.145	0.207	0.199	0.213	0.189	0.303	0.083	0.081	0.079	0.077	0.066
SR	0.164	0.162	0.068	0.045	-0.308	0.490	0.440	0.330	0.240	0.190	0.340	0.320	0.220	0.170	-0.150

Table 7: Daily volatility and return spillover results for category one data

In Table 7 we apply the Diebold-Yilmaz (2012) framework based on the general forecast variance decomposition introduced by Pesaran and Shin (1998) to construct volatility and return spillovers for the respective category one data: 07/10/2017 – 13/10/2022. Specifically, consider a covariance stationary N-variable VAR(p), $x_t = \sum_{i=1}^{p} \phi_i x_{t-i} + \varepsilon_t,\ \ where\ \varepsilon \sim (0,\Sigma)$ is a vector of i.i.d. disturbances. The moving average representation is given by $x_t = \sum_{i=1}^{\infty} A_i \varepsilon_{t-i}$, where for the NxN coefficient matrices $A_i = 0\ for\ i < 0$. We compute the fraction of the H-step-ahead error variance in x_i forecasts due to shocks to $x_j, \forall\ j \neq i$ for each i. The directional volatility (return) spillover received by asset i *from* all other assets j is written as: $S_{i.}^{g}(H) = \frac{\sum_{1 \le j \le N, j \neq i} \widetilde{\theta_{ij}}(H)}{N} * 100$. Similarly, the directional volatility (return) spill-overs from asset j *to* all other assets i may be written as: $S_{.i}^{g}(H) = \frac{\sum_{1 \le j \le N, j \neq i} \widetilde{\theta_{ji}}(H)}{N} * 100$. Accordingly, net spillover from asset i to all other assets may be written as $S_{i}^{g}(H) = S_{i}^{g}(H) - S_{i.}^{g}(H)$..In the table 'from' and 'to' others provides directional spillovers in percentage. Net spill over is calculated from 'to' minus 'from' others results. Panel A presents daily volatility and respective net spillovers, while Panel B provides corresponding results for expected returns

Panel A: Volatility Spillover (%)									
Asset	Bonds	EUA	Commodities	Currencies	Equities	Lumber	REITS	from others	Net Spillover
Bonds	13.976	0.040	0.081	0.030	0.035	0.054	0.070	0.309	-0.026
EUA	0.087	13.784	0.023	0.234	0.057	0.046	0.055	0.502	0.093
Commodities	0.060	0.345	13.734	0.008	0.048	0.044	0.047	0.552	-0.149
Currencies	0.015	0.051	0.007	14.174	0.019	0.005	0.015	0.112	0.188
Equities	0.068	0.023	0.126	0.004	14.008	0.028	0.029	0.278	-0.080
Lumber	0.028	0.032	0.120	0.009	0.014	14.074	0.008	0.212	0.025
REITS	0.026	0.103	0.046	0.015	0.025	0.060	14.011	0.275	-0.051
to others (spill over)	0.283	0.595	0.403	0.300	0.198	0.236	0.224	2.239	
to others (incl. own)	14.259	14.379	14.137	14.474	14.206	14.310	14.235	100.000	
Panel B: Return Spillover (%)									
Bonds	11.378	0.162	0.567	0.711	0.896	0.173	0.400	2.908	-0.819
EUA	0.121	12.088	0.476	0.226	0.676	0.281	0.417	2.198	-0.681
Commodities	0.542	0.436	10.266	0.232	1.631	0.268	0.911	4.020	-0.228
Currencies	0.477	0.198	0.611	10.185	1.183	0.268	1.363	4.101	-2.058
Equities	0.544	0.320	1.108	0.329	7.480	0.359	4.146	6.806	2.975
Lumber	0.163	0.160	0.285	0.105	0.705	12.267	0.600	2.018	-0.333
REITS	0.243	0.241	0.745	0.440	4.690	0.336	7.591	6.694	1.143
to others (spill over)	2.089	1.517	3.792	2.043	9.781	1.686	7.837	28.745	
to others (incl. own)	13.467	13.605	14.058	12.228	17.261	13.953	15.429	100.000	

Table 8: Daily volatility and return spillover results for category two data

In Table 8 we apply the Diebold and Yilmaz (2012) framework based on the general forecast variance decomposition introduced by Pesaran and Shin (1998) to construct volatility and return spillovers for the respective category two data: 01/03/2021 – 13/10/2022. Specifically, consider a covariance stationary N-variable VAR(p), $x_t = \sum_{i=1}^{p} \phi_i x_{t-i} + \varepsilon_t$, $where\ \varepsilon \sim (0,\Sigma)$ is a vector of i.i.d. disturbances. The moving average representation is given by $x_t = \sum_{i=1}^{\infty} A_i \varepsilon_{t-i}$, where for the NxN coefficient matrices $A_i = 0\ for\ i < 0$. We compute the fraction of the H-step-ahead error variance in x_i forecasts due to shocks to $x_j, \forall\ j \neq i$ for each i. The directional volatility (return) spillover received by asset i *from* all other assets j is written as: $S_{i.}^{g}(H) = \frac{\sum_{1\le j\le N, j\neq i} \widetilde{\theta_{ij}}(H)}{N} * 100$. Similarly, the directional volatility (return) spill-overs from asset j *to* all other assets i may be written as: $S_{.i}^{g}(H) = \frac{\sum_{1\le j\le N, j\neq i} \widetilde{\theta_{ji}}(H)}{N} * 100$. Accordingly, net spillover from asset i to all other assets may be written as $S_{i}^{g}(H) = S_{.i}^{g}(H) - S_{i.}^{g}(H)$. In the table 'from' and 'to' others provides directional spillovers in percentage. Net spill over is calculated from 'to' minus 'from' others results. Panel A presents daily volatility and respective net spillovers, while Panel B provides corresponding results for expected returns

Panel A: Volatility Spillover (%)										
Asset	Bonds	EUA	Commodities	Currencies	Equities	GEO	Lumber	REITS	from others	Net Spillover
Bonds	11.816	0.004	0.095	0.055	0.048	0.352	0.021	0.108	0.684	2.105
EUA	0.014	12.248	0.032	0.099	0.052	0.029	0.006	0.021	0.252	0.271
Commodities	0.039	0.263	12.065	0.007	0.007	0.005	0.069	0.046	0.435	-0.116
Currencies	0.055	0.066	0.001	12.307	0.010	0.024	0.003	0.035	0.193	0.056
Equities	0.049	0.041	0.022	0.007	12.158	0.063	0.018	0.142	0.342	0.051
GEO	2.506	0.108	0.018	0.033	0.095	9.534	0.064	0.141	2.966	-2.417
Lumber	0.044	0.010	0.108	0.024	0.045	0.009	12.245	0.015	0.255	-0.006
REITS	0.083	0.031	0.044	0.025	0.136	0.067	0.068	12.047	0.453	0.055
to others (spill over)	2.789	0.523	0.320	0.249	0.393	0.549	0.249	0.508	5.582	
to others (incl. own)	14.605	12.771	12.384	12.556	12.551	10.083	12.494	12.555	100.000	
Panel B: Return Spillover (%)										
Bonds	10.681	0.311	0.154	0.567	0.121	0.186	0.055	0.426	1.819	-0.589
EUA	0.346	11.255	0.060	0.303	0.232	0.051	0.118	0.135	1.245	-0.119
Commodities	0.103	0.137	11.035	0.467	0.347	0.064	0.174	0.175	1.465	-0.435
Currencies	0.421	0.165	0.332	8.100	1.758	0.023	0.237	1.463	4.400	-0.362
Equities	0.003	0.144	0.207	1.214	6.625	0.023	0.277	4.006	5.875	1.353
GEO	0.178	0.216	0.064	0.099	0.197	11.560	0.032	0.155	0.940	-0.561
Lumber	0.018	0.055	0.109	0.328	0.479	0.011	11.220	0.280	1.280	-0.213
REITS	0.161	0.098	0.103	1.060	4.095	0.022	0.175	6.785	5.715	0.924
to others (spill over)	1.230	1.127	1.031	4.038	7.228	0.380	1.066	6.639	22.739	
to others (incl. own)	11.911	12.381	12.065	12.138	13.853	11.939	12.287	13.424	100.000	

Table 9: Daily volatility and return spillover results for category three data

In Table 9 we apply the Diebold and Yilmaz (2012) framework based on the general forecast variance decomposition introduced by Pesaran and Shin (1998) to construct volatility and return spillovers for the respective category two data: 04/08/2021 – 13/10/2022. Specifically, consider a covariance stationary N-variable VAR(p), $x_t = \sum_{i=1}^{p} \phi_i x_{t-i} + \varepsilon_t$, $where\ \varepsilon \sim (0,\Sigma)$ is a vector of i.i.d. disturbances. The moving average representation is given by $x_t = \sum_{i=1}^{\infty} A_i \varepsilon_{t-i}$, where for the NxN coefficient matrices $A_i = 0\ for\ i < 0$. We compute the fraction of the H-step-ahead error variance in x_i forecasts due to shocks to $x_j, \forall\ j \neq i$ for each i. The directional volatility (return) spillover received by asset i *from* all other assets j is written as: $S_{i.}^{g}(H) = \frac{\sum_{1\le j\le N, j\neq i} \widetilde{\theta_{ij}}(H)}{N} * 100$. Similarly, the directional volatility (return) spill-overs from asset j *to* all other assets i may be written as: $S_{.i}^{g}(H) = \frac{\sum_{1\le j\le N, j\neq i} \widetilde{\theta_{ji}}(H)}{N} * 100$. Accordingly, net spillover from asset i to all other assets may be written as $S_{i}^{g}(H) = S_{.i}^{g}(H) - S_{i.}^{g}(H)$. In the table 'from' and 'to' others provides directional spillovers in percentage. Net spill over is calculated from 'to' minus 'from' others results. Panel A presents daily volatility and respective net spillovers, while Panel B provides corresponding results for expected returns

Asset	Bonds	EUA	Commodities	Currencies	Equities	GEO	Lumber	NGEO	REITS	from others	Net Spillover
Panel A: Volatility Spillover (%)											
Bonds	10.283	0.005	0.078	0.061	0.050	0.313	0.021	0.208	0.093	0.828	2.338
EUA	0.018	10.655	0.030	0.086	0.070	0.028	0.008	0.201	0.014	0.456	0.315
Commodities	0.012	0.056	9.755	0.016	0.014	0.042	0.130	0.770	0.315	1.356	-0.532
Currencies	0.067	0.064	0.009	10.802	0.012	0.025	0.003	0.083	0.046	0.309	0.014
Equities	0.064	0.047	0.034	0.010	10.600	0.061	0.015	0.017	0.263	0.512	0.208
GEO	2.470	0.079	.052	0.039	0.085	8.115	0.058	0.039	0.174	2.996	-2.421
Lumber	0.037	0.010	0.166	0.022	0.059	0.004	10.668	0.125	0.020	0.444	0.236
NGEO	0.421	0.490	0.105	0.052	0.183	0.034	0.238	9.586	0.003	1.525	-1.374
REITS	0.076	0.021	0.350	0.037	0.246	0.067	0.206	0.151	9.957	1.154	-0.226
to others (spill over)	3.166	0.772	0.824	0.322	0.720	0.574	0.680	1.594	0.928	9.580	
to others (incl. own)	13.449	11.426	10.579	11.125	11.319	8.690	11.348	11.179	10.885	100.000	
Panel B: Return Spillover (%)											
Bonds	9.309	0.307	0.081	0.577	0.126	0.176	0.074	0.025	0.436	1.802	-0.596
EUA	0.354	9.929	0.063	0.239	0.163	0.083	0.147	0.040	0.092	1.182	0.603
Commodities	0.070	0.175	9.133	0.473	0.371	0.054	0.201	0.394	0.241	1.978	-0.534
Currencies	0.428	0.117	0.340	7.038	1.531	0.001	0.288	0.050	1.318	4.073	-0.154
Equities	0.003	0.105	0.221	1.031	5.771	0.047	0.342	0.037	3.554	5.340	1.527
GEO	0.154	0.218	0.051	0.130	0.173	8.824	0.027	1.431	0.103	2.287	-0.194
Lumber	0.013	0.092	0.181	0.393	0.635	0.025	9.223	0.033	0.517	1.888	-0.500
NGEO	0.024	0.685	0.365	0.133	0.309	1.640	0.022	7.625	0.309	3.487	-3.441
REITS	0.161	0.086	0.142	0.943	3.559	0.067	0.288	0.045	5.820	5.292	1.278
to others (spill over)	1.206	1.785	1.444	3.920	6.867	2.093	1.388	2.056	6.569	27.329	
to others (incl. own)	10.515	11.714	10.577	10.957	12.638	10.917	10.612	9.681	12.389	100.000	

Table 10: Portfolio strategy definitions for in-sample-out-of-time performance tests
Portfolio strategy definitions for in-sample-out-of-time performance tests

Portfolio	Description	Period
	Category 1	
Benchmark + C	Benchmark + EUA	07 Oct 2017 – 13 Oct 2022
Benchmark	Bonds, Commodities, Currencies, Equities, Lumber, REITS	
	Category 2	**Period**
Benchmark + G	Benchmark + GEO	01 March 2021 – 13 Oct 2022
Benchmark + GC	Benchmark + GEO + EUA	
Benchmark	Bonds, Commodities, Currencies, Equities, Lumber, REITS	
	Category 3	**Period**
Benchmark + N	Benchmark + NGEO	04 August 2021 – 13 Oct 2022
Benchmark + NC	Benchmark + NGEO + EUA	
Benchmark + NG	Benchmark + NGEO + GEO	
Benchmark + NGC	Benchmark + NGEO + GEO + EUA	
Benchmark	Bonds, Commodities, Currencies, Equities, Lumber, REITS	

Table 11: Test results for differences in in-sample-out-of-time portfolio Sharpe Ratios for category one data

Table 11 presents in-sample-out-of-time performance results for difference in Sharpe Ratios for Naïve Diversification, Long, Long-Short and the Minimum Volatility strategies using category one data. We report average in-sample out-of-time (t+5, t+10, t+20) Sharpe Ratios from return bootstraps on the Benchmark + C portfolio and the Benchmark and the difference (Diff) thereof. The test algorithm is extensively outlined in Ledoit-Wolf (2008) and is as follows. We generate a series of portfolio returns on the basis on a 5, 10 and 20-day holding periods using optimal weights generated using the prior 90 days asset price history. Hence the ith difference between generated Sharpe Ratios for portfolios of the same strategy as for all portfolio pairs (j,k) is: $\widehat{\Delta}_i = \widehat{SR}_{j_i} - \widehat{SR}_{k_i}$ and construct an inverted studentized bootstrap confidence interval for the difference in the Sharpe Ratios. We conclude that $\widehat{\Delta}_i$ is statistically different from zero if zero is not contained in the obtained interval. We generate stationary block bootstraps in a Politis and Romano (1992) sense and resample blocks of pairs from the observed pairs $(r_{tj}, r_{tk})'$ $t = 1, \ldots, T$ with replacement using optimal block samples for return pair series generated above. We generate 10,000 bootstrap samples and test $H_0: \Delta_{SharpeRatio} = 0$ applying the decision criteria outlined above.

Strategy	Holding Period (days)	Benchmark + C	Benchmark	Diff	
	5	0.318	0.130	0.188	***
Naive Diversification	10	0.457	0.194	0.263	***
	20	0.652	0.275	0.377	***
	5	0.459	0.244	0.215	***
Long	10	0.600	0.303	0.296	***
	20	0.731	0.295	0.436	***
	5	0.104	0.019	0.085	
Long-Short	10	0.175	0.047	0.129	
	20	0.258	-0.038	0.296	***
	5	0.299	0.179	0.119	***
Minimum Volatility	10	0.428	0.236	0.192	***
	20	0.628	0.359	0.269	***

Table 12: Test results for differences in in-sample-out-of-time portfolio Sharpe Ratios for category two data

Table 12 presents in-sample-out-of-time performance results for difference in Sharpe Ratios for Naïve Diversification, Long, Long-Short and the Minimum Volatility strategies using category two data. We report average in-sample out-of-time (t+5, t+10, t+20) Sharpe Ratios from return bootstraps on the Benchmark + GC, Benchmark + G, the Benchmark portfolio and their respective differences to the Benchmark portfolio (Diff). The test algorithm is extensively outlined in Ledoit-Wolf (2008) and is as follows. We generate a series of portfolio returns on the basis on a 5, 10 and 20-day holding periods using optimal weights generated using the prior 90 days asset price history. Hence the ith difference between generated Sharpe Ratios for portfolios of the same strategy as for all portfolio pairs (j,k) is: $\widehat{\Delta}_i = \widehat{SR}_{j_i} - \widehat{SR}_{k_i}$ and construct an inverted studentized bootstrap confidence interval for the difference in the Sharpe Ratios. $\widehat{\Delta}_i$ is statistically different from zero if zero is not contained in the obtained interval. We generate stationary block bootstraps in a Politis and Romano (1992) sense and resample blocks of pairs from the observed pairs $(r_{tj}, r_{tk})'$ $t = 1, \ldots, T$ with replacement using optimal block samples for return pair series generated above. We generate 10,000 bootstrap samples and test $H_0 \colon \Delta_{SharpeRatio} = 0$ applying the decision criteria outlined above.

Strategy	Holding Period (days)	Benchmark + G	Diff		Benchmark + GC	Diff		Benchmark
	5	-0.078	0.098		0.000	0.176		-0.176
Naive Diversification	10	-0.085	0.156		0.025	0.266		-0.240
	20	-0.101	0.291		0.052	0.444		-0.392
	5	0.706	0.608	***	0.682	0.584	***	0.097
Long	10	1.185	0.874	***	1.107	0.795	***	0.311
	20	1.558	1.080	***	1.374	0.896	***	0.478
	5	0.874	0.846	***	1.799	1.771	***	0.028
Long-Short	10	1.405	1.210	***	1.245	1.050	***	0.195
	20	2.121	1.756	***	0.758	0.393	***	0.365
	5	0.092	0.022		0.049	-0.021		0.070
Minimum Volatility	10	0.172	0.042		0.102	-0.027		0.129
	20	0.237	0.024		0.184	-0.029		0.212

Table 13: Test results for differences in in-sample-out-of-time portfolio Sharpe Ratios for category three data

Table 13 presents in-sample-out-of-time performance results for difference in Sharpe Ratios for Naïve Diversification, Long, Long-Short and the Minimum Volatility strategies using category three data. We report average in-sample out-of-time (t+5, t+10, t+20) Sharpe Ratios from return bootstraps on portfolios that include NGEO, GEO and EUA and their respective differences to the Benchmark portfolio (Diff). We generate a series of portfolio returns on the basis on a 5, 10 and 20-day holding periods using optimal weights generated using the prior 90 days asset price history. Hence the ith difference between generated Sharpe Ratios for portfolios of the same strategy as for all portfolio pairs (j, k) is $\widehat{\Delta}_i = \widehat{SR}_{j_i} - \widehat{SR}_{k_i}$ and construct an inverted studentized bootstrap confidence interval for the difference in the Sharpe Ratios. $\widehat{\Delta}_i$ is statistically different from zero if zero is not contained in the obtained interval. We generate stationary block bootstraps in a Politis and Romano (1992) sense and resample blocks of pairs from the observed pairs $(r_{tj}, r_{tk})'$ $t = 1, \ldots, T$ with replacement using optimal block samples for return pair series generated above. We generate 10,000 bootstrap samples and test H_0: $\Delta_{SharpeRatio} = 0$ using the decision criteria above.

Strategy	Holding period (days)	Benchmark + N	Diff		Benchmark + NC	Diff		Benchmark + NG	Diff		Benchmark + NGC	Diff		Benchmark
Naive Diversification	5	-0.805	-0.187		-0.790	-0.172		-0.874	-0.256	**	-0.842	-0.224		-0.618
	10	-1.342	-0.292	**	-1.234	-0.184		-1.393	-0.343	**	-1.269	-0.219		-1.050
	20	-2.679	-0.624	***	-2.314	-0.259		-2.659	-0.604	***	-2.357	-0.302	**	-2.055
Long	5	0.167	0.090		0.001	-0.076		0.168	0.091		0.147	0.070		0.077
	10	0.166	0.027		0.039	-0.100		0.274	0.135		0.323	0.184		0.139
	20	-0.256	-0.261		-0.307	-0.313		0.081	0.076		0.249	0.244		0.005
Long-Short	5	0.846	0.300	***	0.661	0.115		0.836	0.290	***	0.742	0.196	***	0.546
	10	1.359	0.413	***	1.057	0.110	**	1.252	0.305	***	1.162	0.215	***	0.946
	20	1.644	0.288	***	1.258	-0.097		1.783	0.428	***	1.625	0.270	***	1.355
Minimum Volatility	5	-0.500	-0.287	***	-0.671	-0.458	***	-0.748	-0.535	***	-0.860	-0.647	***	-0.213
	10	-0.873	-0.476	***	-1.130	-0.733	***	-1.278	-0.880	***	-1.388	-0.990	***	-0.398
	20	-1.658	-0.919	***	-2.020	-1.281	***	-2.198	-1.459	***	-2.348	-1.608	***	-0.740

Appendix Tables

Table A1: Summary statistics for in-sample out-of-time rolling weights for category one, two and three data

The table provides in-sample out-of-time summary statistics for rolling weights for Benchmark + C (category one data), Benchmark + G (category two data) and Benchmark + N (category three data) for each strategy. Portfolio descriptions are presented in Table 10. Max and Min report the maximum and minimum weights in the respective series while Abs Mean reports the mean of the absolute optimal weight series. The column EUA represents the statistics from the rolling optimal in-sample out-of-time weights from the Benchmark + C portfolio using category one data. The column GEO is respectively from Benchmark + G using category two data. The column NGEO represents Benchmark + N using category three data. The algorithm works as follows. Using data from t-90 days, we estimate optimal weights per portfolio and hold these positions for t+20 days. The algorithm then shifts by a day. Consequently, we obtain a time series of length T-90 from which we calculate summary statistics for the columns EUA, GEO and NGEO.

Strategy		EUA	GEO	NGEO
	Max	0.143	0.143	0.143
	Min	0.143	0.143	0.143
Naive Diversification	Abs Mean	0.143	0.143	0.143
	Max	0.305	0.237	0.223
	Min	0.000	0.000	0.000
Long	Abs Mean	0.160	0.131	0.081
	Max	0.305	0.238	0.223
	Min	-1.000	-0.490	-0.538
Long-Short	Abs Mean	0.266	0.255	0.242
	Max	0.138	0.109	0.113
	Min	0.000	0.024	0.056
Minimum Volatility	Abs Mean	0.057	0.060	0.088

Appendix Figures

Figure A1: Rolling 250-day rolling return correlations between traditional assets and European Emission Allowances (EUA)

The figure shows rolling 250-day correlation for category one data (07/10/2017 – 13/10/2022). The horizontal axis shows years while the vertical axis shows the Bravais-Pearson correlation coefficient (./100) of all assets with EUA.

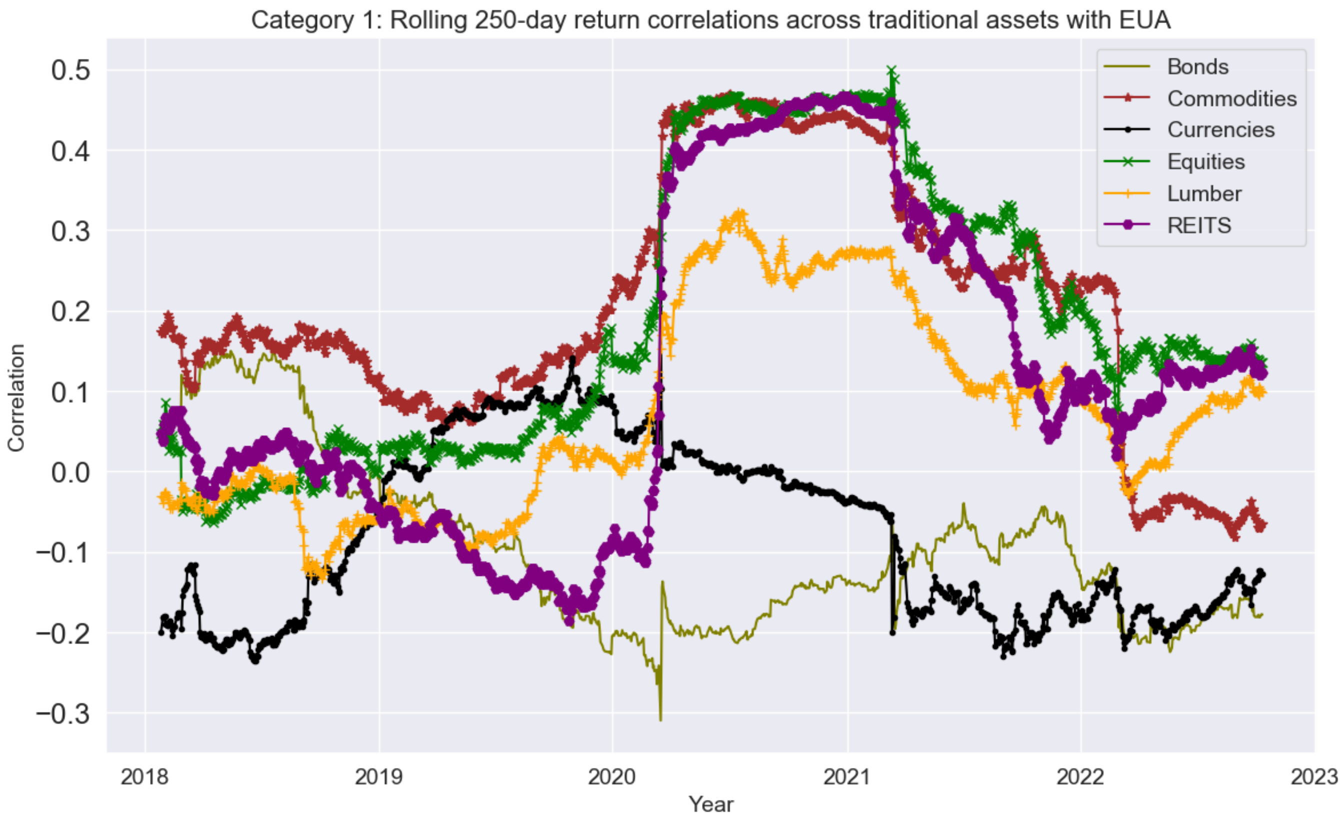


Figure A2: Rolling 60-day rolling correlation between traditional assets with GEO
The figure shows rolling 60-day correlation for category two data (01/03/2022 – 13/10/2022). The horizontal axis shows date measured in year and month while the vertical axis shows the Bravais-Pearson correlation coefficient (./100) of all assets with GEO.

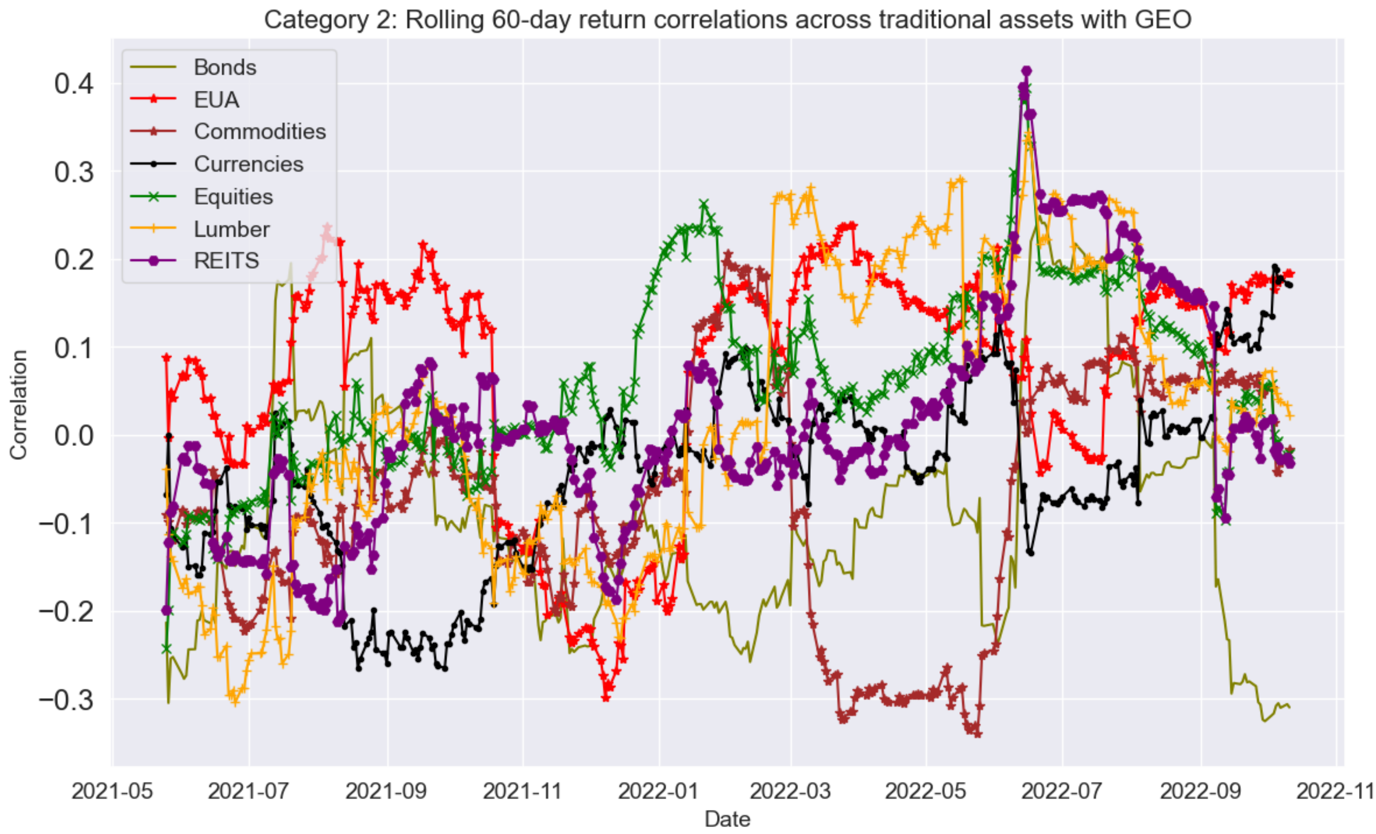


Figure A3: Rolling 60-day rolling correlation between traditional assets with NGEO
The figure shows rolling 60-day correlation for category three data (04/08/2021 – 13/10/2022). The horizontal axis shows the date measured in year and month while the vertical axis shows the Bravais-Pearson correlation coefficient (./100) of all assets with NGEO.

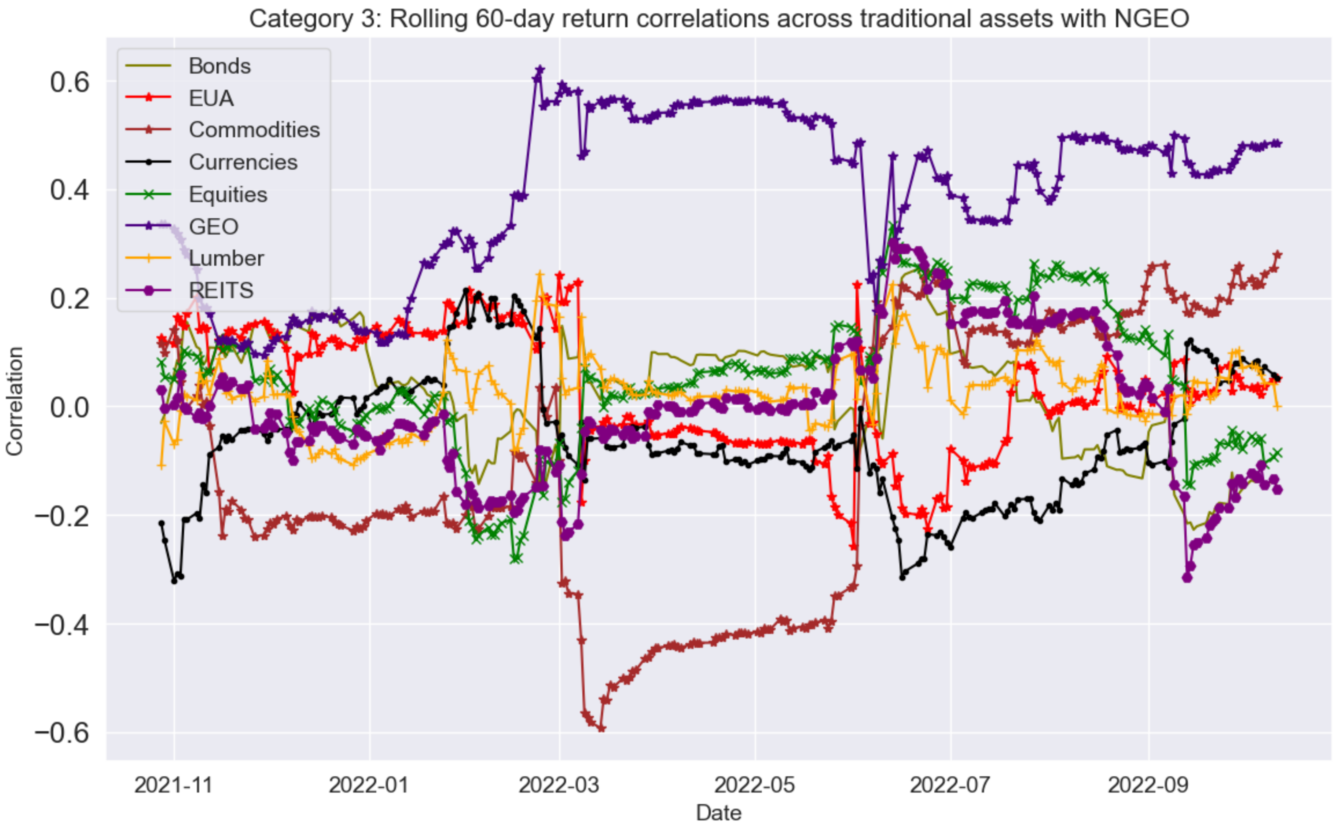
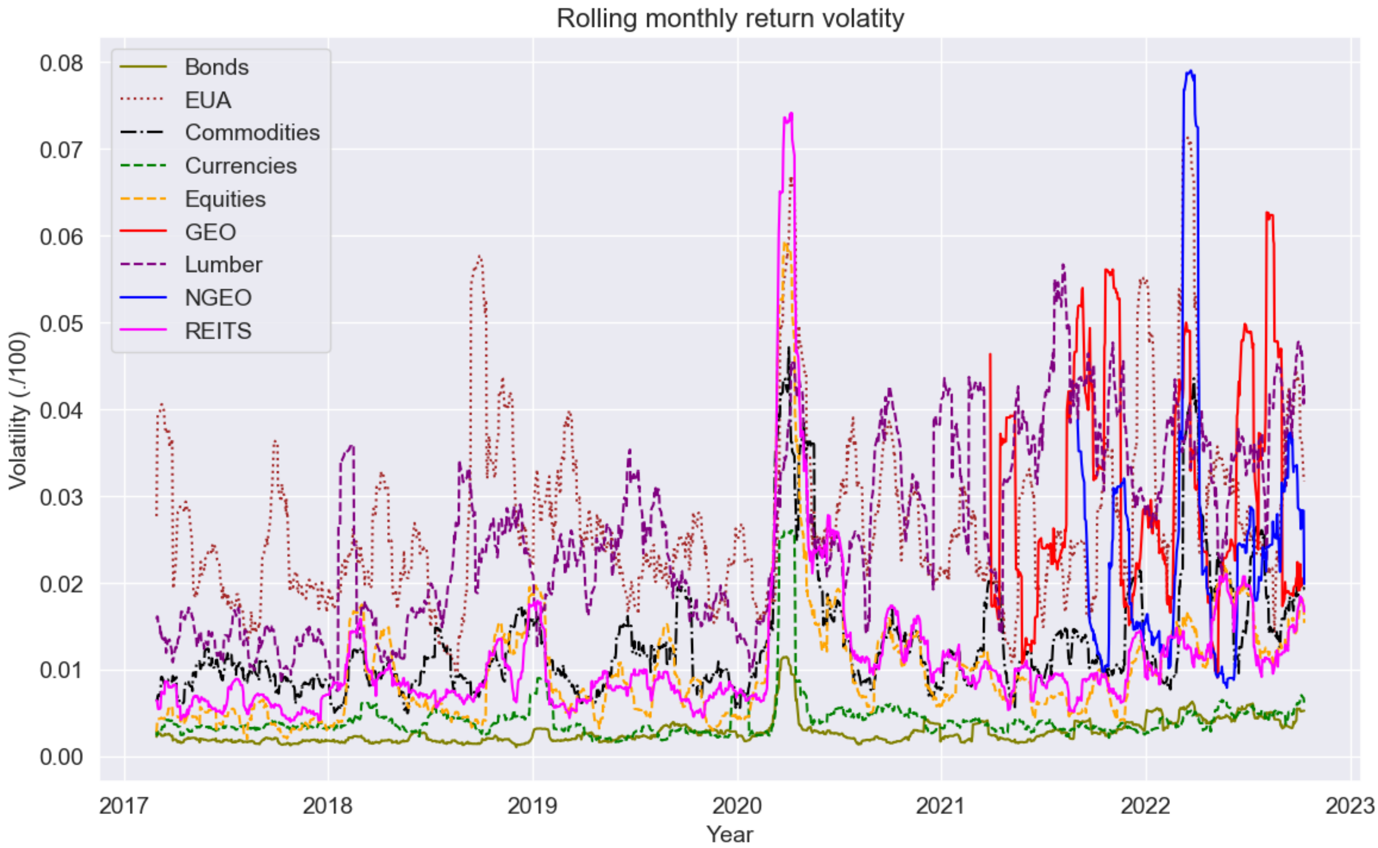


Figure A4: Rolling 20-day volatility for all assets
The figure shows rolling 20-day volatility for all assets. The horizontla axis shows the Year. The vertical axis shows Volatility (./100)



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